

NAVAL POSTGRADUATE SCHOOL

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THESIS

**IDENTIFICATION OF
PUSH-TO-TALK TRANSMITTERS
USING WAVELETS**

by

Yalçın Payal

December, 1995

Thesis Advisors :

Ralph Hippenstiel
Monique P. Fargues

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**IDENTIFICATION OF
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USING WAVELETS**

Yalçın Payal
Lieutenant Junior Grade, Turkish Navy
B.S., Turkish Naval Academy, 1989

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of the requirements for the degree of

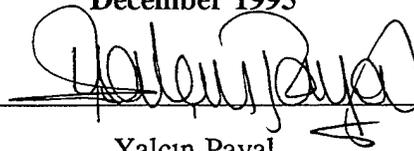
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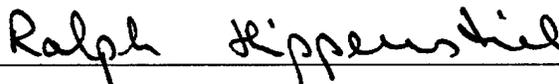
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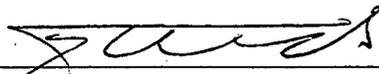


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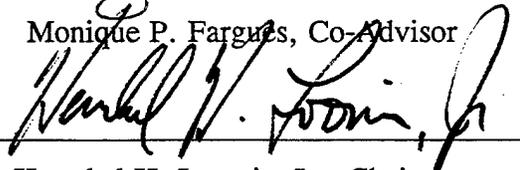
Approved by:



Ralph Hippenstiel, Thesis Co-Advisor



Monique P. Fargues, Co-Advisor



Herschel H. Loomis, Jr., Chairman

Department of Electrical and Computer Engineering

ABSTRACT

The main objective of this study is to find a wavelet-based, feature extracting algorithm for push-to-talk transmitter identification. A distance-measure algorithm is introduced to classify signals belonging to one of four transmitters. The signals are first preprocessed to put them into a form suitable for wavelet analysis. The preprocessing scheme includes taking envelopes and differentials. Median filtering is also applied to the envelopes and the differentials to denoise the data. The preprocessed data takes on a pulse-like shape, which is suitable for wavelet processing. The distance algorithm is applied to the outputs (scales) of the wavelet transform. The distance algorithm uses local extrema of the wavelet coefficients, and computes the distance between the local extrema of a template and the processed signals. A small distance implies high similarity. A signal from each transmitter is selected as a template. The distance measure is computed between any signal of interest and the reference templates. The signals are identified to belong to one of four transmitters according to the distance measure. A small distance measure indicates that the signal belongs to the transmitter from which the template originated. The distance algorithm can classify correctly the four different signal sets provided for the research and, even at lower signal-to-noise levels, good identification is achieved.

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I. INTRODUCTION

Signals produced by different transmitters have different characteristics. The transient response, obtained when the transmitter is turned on or off, is basically a build-up from off-state to on-state, or vice versa. These responses are unique to the specific transmitter and differ even among units of the same make and model. This observation applies only to the turn-on response. Therefore, a technique to extract the uniqueness of the turn-on transients will help to identify the transmitter.

Time-frequency analysis of stationary signals is a well-known subject. The Fourier Transform method is quite efficient for this kind of analysis. However, when the signal of interest is non-stationary, the Fourier Transform method is not appropriate, because it uses as a basis function the complex exponential that expands over infinite time. That is, it lacks the time-evolution of frequencies that is very important for the analysis of transients of short duration.

A sliding time-window was introduced by Gabor (1946) to gain time information from the Fourier Transform method. This modified Fourier Transform method is called the Short-Time Fourier Transform (STFT), and uses a modulated window as a basis function. The STFT looks through the window, and processes just a small portion of the signal and, by sliding the window, processes another small portion of the signal successively. Thus, once a window has been chosen, time-frequency resolution is fixed. The problem with this method is that the signal must be stationary within these small portions, or else the method will have limitations in reflecting the time-evolution of frequencies within the window.

Another technique for transient signal analysis is the Wavelet Transform (WT), which seems more efficient in terms of time evolution of frequencies than the STFT. Basis functions of the WT, unlike the complex exponential of the Fourier Transform,

do not have infinite duration. They are nonzero for only a short duration. Accordingly, this “compact support” makes the WT localized, not only in frequency but in time as well. Moreover, wavelets provide the flexibility to choose the particular wavelet function that is appropriate for a specific application. This is possible since there are a large number of compactly supported wavelets that can be used as orthogonal basis functions.

The purpose of this thesis is to investigate the use of the Wavelet Analysis for the classification of transient signals of push-to-talk transmitters, and to find an algorithm to identify transmitters. Chapter II gives a brief introduction to wavelet analysis and median filtering. Chapter III explains the algorithms to identify the push-to-talk transmitters, and Chapter IV shows the results and presents the recommendations for future studies.

II. SIGNAL DECOMPOSITION AND MEDIAN FILTERING

A. CONCEPT OF A BASIS

A linear independent set S of vectors $\{\mathbf{v}_k\}$ forms a basis of the space \mathbf{V} if every element $\mathbf{x} \in \mathbf{V}$ can be represented in the form of a series

$$\mathbf{x} = \sum_k c_k \mathbf{v}_k, \quad (2.1)$$

where c_k 's are the coefficients.

A set S of vectors are said to be linearly independent if and only if there are numbers $c_1, c_2, \dots, c_n, \dots$, all of which are zeros, such that

$$c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \dots + c_n \mathbf{v}_n + \dots = 0.$$

B. INNER PRODUCT SPACES

An inner product space is a vector space \mathbf{V} with an inner product defined on it. If $\mathbf{x} = (x_1, x_2, \dots, x_n)$ and $\mathbf{y} = (y_1, y_2, \dots, y_n)$ are vectors in \mathbf{V} , then the inner product of \mathbf{x} and \mathbf{y} are defined by

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \cdot \mathbf{y}^* = x_1 \cdot y_1^* + x_2 \cdot y_2^* + \dots + x_n \cdot y_n^*. \quad (2.2)$$

Thus, an inner product of \mathbf{x} and \mathbf{y} is a mapping from $\mathbf{x} \cdot \mathbf{y}$, cartesian product space, into the scalar product field.

The inner product has the following properties:

Let $\mathbf{x}, \mathbf{y}, \mathbf{z}$ be vectors in \mathbf{V} , and c any real-valued scalar, then

1. $\langle (\mathbf{x} + \mathbf{y}), \mathbf{z} \rangle = \langle \mathbf{x}, \mathbf{z} \rangle + \langle \mathbf{y}, \mathbf{z} \rangle$.
2. $\langle \mathbf{x}, \mathbf{y} \rangle = \langle \mathbf{y}, \mathbf{x} \rangle$.
3. $\langle c \cdot \mathbf{x}, \mathbf{y} \rangle = c \cdot \langle \mathbf{x}, \mathbf{y} \rangle$.
4. $\langle \mathbf{x}, \mathbf{x} \rangle \geq 0$ with equality if and only if $\mathbf{x} = 0$.

Functions like $f(x)$, on the interval $0 \leq x \leq X$, can be thought of as a vector with a continuum of components along the whole interval. In this case, the summation

can be replaced by integration (Burrus and Gopinath, 1993) to find the length of the vector:

$$\| f \|^2 = \int_0^X |f(x)|^2 dx \quad (2.3)$$

Inner products of two functions $f(x)$ and $g(x)$ also can be produced in the same manner:

$$\langle f(x), g(x) \rangle = f^T \cdot g^* = \int_0^X f(x) \cdot g^*(x) dx,$$

where $f(x)$ and $g(x)$ are defined between $0 \leq x \leq X$.

C. ORTHONORMAL BASES

Any two vectors are called orthogonal if they are perpendicular to each other. A basis V_1, V_2, \dots, V_n is called orthonormal if all the elements of V_1, V_2, \dots, V_n are orthogonal to each other, and $\|V_i\| = 1, i = 1, 2, \dots, n$ (Strang, 1976).

V_1, V_2, \dots, V_n is said to form an orthonormal basis set if the inner product is

$$\langle V_i, V_j \rangle = \begin{cases} 0 & \text{when } i \neq j \\ 1 & \text{when } i = j. \end{cases}$$

D. FOURIER TRANSFORM

The Fourier Transform of a square integrable function $f(t)$ is defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = \langle f(t), e^{j\omega t} \rangle, \quad (2.4)$$

which is called the Fourier Analysis Formula. The inverse Fourier Transform is given by

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega, \quad (2.5)$$

which is the Synthesis Formula.

E. SAMPLING

The Sampling Theorem allows us to convert the analog signal to a discrete form for discrete signal processing. The main idea is to take samples from a signal at such a rate that the sequence of samples uniquely defines the original analog signal.

Let $f_T(t)$ denote the signal obtained by weighting the periodic sequence of delta functions spaced T seconds apart by the sequence of numbers $\{f_T(t)\}$; then, the sampled signal is

$$f_T(t) = \sum_{n=-\infty}^{\infty} f(nT) \delta(t - nT). \quad (2.6)$$

F. DISCRETE FOURIER TRANSFORM

The Discrete Fourier Transform of a square summable function $f(n)$, $n = 0, 1, 2, \dots, N$, is defined by

$$F(k) = \sum_{n=0}^{N-1} f(n) e^{-j \frac{2\pi}{N} kn}. \quad (2.7)$$

Its inverse is given by

$$f(n) = \frac{1}{N} \sum_{k=0}^{N-1} F(k) e^{j \frac{2\pi}{N} kn}. \quad (2.8)$$

G. SHORT TIME FOURIER TRANSFORM (STFT)

1. Introduction

It can be seen from Equation (2.4) that the Fourier Transform of a signal does not reflect the time-evolution of the frequencies (Vetterli and Kovacevic, 1995). Because it does not show how the frequencies vary with time, it is not possible to extract the transient anomalies in the signal, which is spread over all frequencies.

Gabor (1946) proposed a time-frequency localization technique, which is known as the Short Time Fourier Transform (STFT). In this transform, the signal first is

divided into short consecutive segments, and then the Fourier Transform of each segment is computed. The problem with this transform is the poor time resolution if the transient phenomena is shorter than the time window.

2. The Continuous STFT

In order to obtain the STFT of the signal, the standard Fourier Transform is modified by inserting a time window $g(t)$ in Equation (2.4) as follows:

$$F(\omega, \tau) = \int_{-\infty}^{\infty} f(t) g(t - \tau) e^{-j\omega t} dt. \quad (2.9)$$

Equation (2.9) is called the Short Time Fourier Transform (STFT). When the window $g(t)$ is Gaussian, the STFT is called the Gabor Transform.

3. The Discrete STFT

We can obtain the discrete STFT by sampling the continuous STFT, Equation (2.9), on a uniform grid (Akansu and Haddad, 1992) such that

$$\omega = k\omega_0, \quad \tau = n\tau_0.$$

Thus, we obtain the discrete STFT

$$\begin{aligned} F(k, n) &= \int_{-\infty}^{\infty} f(t) g(t - n\tau_0) e^{-jk\omega_0 t} dt \\ &= \langle f(t), e^{jk\omega_0 t} g^*(t - n\tau_0) \rangle. \end{aligned} \quad (2.10)$$

This is the inner product of the signal $f(t)$ and the modulated window $g(t)$. The inner product yields large values when $f(t)$ has sinusoids at or near the frequency $k\omega_0$. The inner product is small at the resolution cell $k\omega_0, n\tau_0$ when $f(t)$ has sinusoids at locations other than $k\omega_0$.

The inner product $F(k, n)$ is calculated for each window location $n\tau_0$. Then the window is shifted to the next location, $(n + 1)\tau_0$, and $F(k, n)$ is calculated for that resolution cell, $k\omega_0, (n + 1)\tau_0$. This procedure generates a two-parameter family $F(k, n)$, which can be plotted on a time-frequency grid as shown in Figure 2.1 (Akansu and Haddad, 1992).

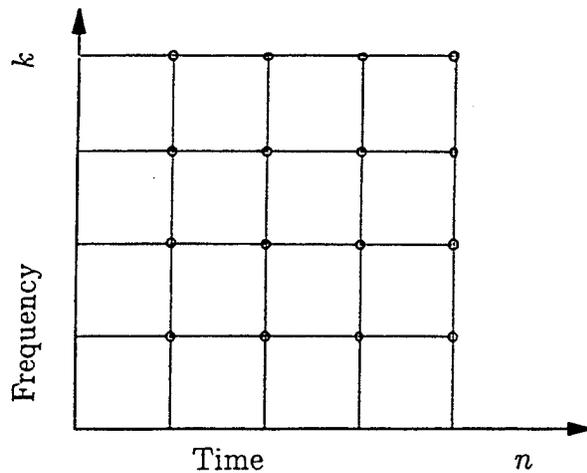


Figure 2.1. Time-frequency grid for the discrete STFT

The STFT, Equation (2.9) can also be interpreted as the convolution of $\tilde{g}(t) \triangleq g(-t)$ with the modulated signal $e^{-j\omega t} f(t)$ such that

$$F(\omega, t) = \tilde{g}(t) * e^{-j\omega t} f(t), \quad (2.11)$$

where $\tilde{g}(t)$ is the time reversal of the window $g(t)$ and “*” denotes the convolution operation. The convolution can be shown schematically as in Figure 2.2.

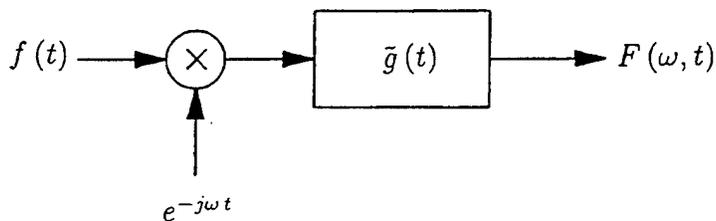


Figure 2.2. Convolution to obtain STFT

If ω_0 in Figure 2.2 is discretized, the modulated filter bank is obtained. If the output of Figure 2.2 is also sampled at $t = n\tau_0$, the corresponding discrete STFT is obtained, as shown in Figure 2.3.

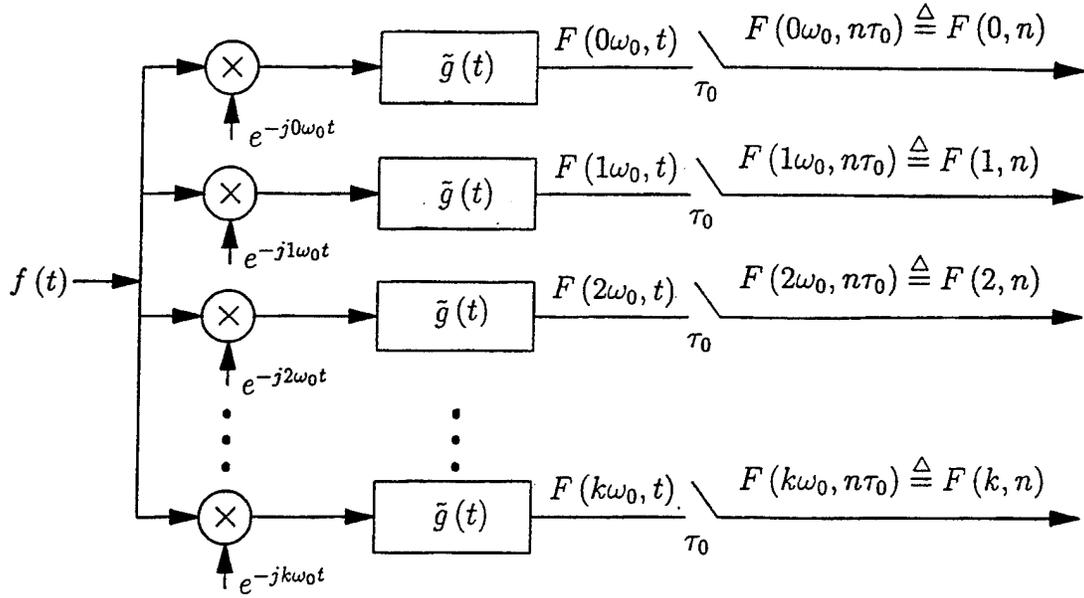


Figure 2.3. The discrete STFT as the modulated filter bank

H. WAVELET TRANSFORM (WT)

The Wavelet Transform (WT) is founded on a set of specific basis functions, which are called “little waves” or wavelets (Young, 1993). They include short duration/high frequency and long duration/low frequency functions. Each element in the wavelet set is constructed from the same function, which is called the “analyzing wavelet” or the “mother wavelet.”

1. Continuous Wavelet Transform (CWT)

The Wavelet Transform of a function $f(t)$, with respect to the mother wavelet $\psi(t)$ is given by

$$\begin{aligned}
 W_f(a, b) &= \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \psi^* \left(\frac{t-b}{a} \right) dt, \quad a \in R^+, \quad b \in R \quad (2.12) \\
 &= \langle f(t), \frac{1}{\sqrt{a}} \psi \left(\frac{t-b}{a} \right) \rangle,
 \end{aligned}$$

where superscript “*” denotes a complex conjugate, and a and b are scaling and translation parameters, respectively. The normalization term $\frac{1}{\sqrt{a}}$ assures that the energy of the scaled mother wavelet is the same as that of the mother wavelet.

There are three conditions for a function to be a mother wavelet. It must be oscillatory, it must decay to zero, and it must integrate to zero (Young, 1993).

As can be seen from the Equation (2.12), the wavelet family incorporates shifted and scaled versions of the mother wavelet, $\psi(t)$. If we define the wavelet family as $\{\psi_{ab}(t)\}_{a,b}$, where

$$\psi_{ab}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right), \quad (2.13)$$

then this enables us to rewrite Equation (2.12) as

$$\begin{aligned} W_f(a,b) &= \int_{-\infty}^{\infty} f(t) \psi_{ab}^*(t) dt \\ &= \langle f(t), \psi_{ab}(t) \rangle. \end{aligned} \quad (2.14)$$

The Continuous Wavelet Transform is invertible, provided that the mother wavelet $\psi(t)$ satisfies the admissibility condition (Young, 1993). A wavelet is admissible if the admissibility constant, C_ψ , satisfies the following inequality

$$C_\psi = \int_0^\infty \frac{|\Psi(\omega)|^2}{\omega} d\omega < \infty, \quad (2.15)$$

where $\Psi(\omega)$ is the Fourier Transform of the wavelet $\psi(t)$.

If the admissibility condition is met for the wavelet function, the inverse CWT exists, and is given by

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_0^\infty \frac{1}{a^2} W_f(a,b) \psi_{ab}(t) da db. \quad (2.16)$$

2. Discrete Wavelet Transform (DWT)

The Discrete Wavelet Transform (DWT) can be obtained by sampling a, b in Equation (2.13) at

$$a = a_0^{-j}, \quad b = kb_0 a_0^{-j}. \quad (2.17)$$

Then the wavelet family takes the form

$$\psi_{jk}(t) = a_0^{j/2} \psi(a_0^j t - kb_0), \quad (2.18)$$

where $j, k \in \mathbf{Z}$. If this wavelet family is complete in $L^2(\mathbf{R})$ for some choice of $\psi(t)$, a , and b , then the set $\{\psi_{jk}\}$ are called frames (Vetterli and Kovacevic, 1995). Now $f(t)$ in $L^2(\mathbf{R})$ can be expressed as

$$f(t) = \sum_j \sum_k d_{jk} \psi_{jk}(t), \quad (2.19)$$

where the wavelet coefficient d_{jk} is the inner product

$$\begin{aligned} d_{jk} &= \langle f(t), \psi_{jk}(t) \rangle \\ &= a_0^{j/2} \int_{-\infty}^{\infty} f(t) \psi^*(a_0^j t - kb_0). \end{aligned} \quad (2.20)$$

Since the sets are frames not bases, they do not satisfy the Parseval's Theorem. Also the expansion, Equation (2.19) is not a unique use of frames.

A family of wavelet functions, $\{\psi_{jk}\}$ is said to be a frame if there exists $0 < A \leq B < \infty$, such that, for all $f(t)$ in $L^2(\mathbf{R})$,

$$A \|f(t)\|^2 \leq \sum_j \sum_k \|\langle \psi_{jk}, f(t) \rangle\|^2 \leq B \|f(t)\|^2, \quad (2.21)$$

where A and B are frame bounds (Vetterli and Kovacevic, 1995).

When the two frame bounds are equal, the frame is called a tight frame. If ψ_{jk} is a tight frame, with a frame bound $A = 1$, and if $\|\psi_{jk}\|^2 = 1$ for all j, k , then ψ_{jk} forms an orthonormal basis (Vetterli and Kovacevic, 1995).

Since ψ_{jk} is a tight frame, Equation (2.21) becomes

$$\sum_j \sum_k \|\langle \psi_{jk}, f(t) \rangle\|^2 = A \|f(t)\|^2. \quad (2.22)$$

The reconstruction formula can be derived from the Equations (2.19), (2.20), and (2.22) as

$$f(t) = \frac{1}{A} \sum_j \sum_k \langle \psi_{jk}, f(t) \rangle \psi_{jk}. \quad (2.23)$$

When $A = 1$, Equation (2.23) is equivalent to Equation (2.19).

Sampling a and b in CWT, Equation (2.13), as stated in Equations (2.17) and (2.18), makes the translation steps, $b = kb_0 a_0^{-j}$, proportional to the time scaling, $a = a_0^{-j}$. Accordingly, large scale values correspond to good time resolution, whereas small scale values correspond to poor time resolution.

The uncertainty principle states that, for a given transform pair, $f(t) \leftrightarrow F(\omega)$, the inequality

$$\sigma_t \sigma_\omega \geq \frac{1}{2} \quad (2.24)$$

holds, where σ_t and σ_ω are defined as follows (Akansu and Haddad, 1992):

$$\sigma_t^2 = \frac{\int t^2 |f(t)|^2 dt}{\int |f(t)|^2 dt}, \quad \sigma_\omega^2 = \frac{\int \omega^2 |F(\omega)|^2 d\omega}{\int |F(\omega)|^2 d\omega},$$

where the integration equation limits $\pm\infty$ are suppressed.

As seen in Equation (2.24), there is a lower bound for the product $\sigma_t \sigma_\omega$; thus, an enhancement in one domain results in degradation in the other domain. Figure 2.4 shows the time-frequency tiling for the WT.

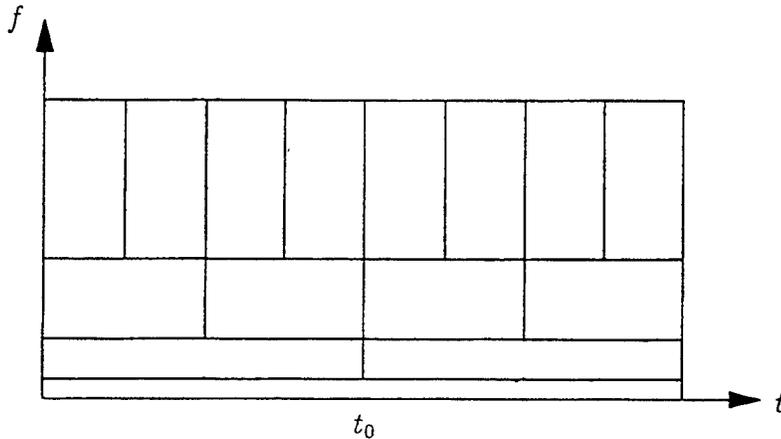


Figure 2.4. Time-frequency tiling for the WT

3. Multiresolution Analysis

a. Multiresolution Spaces

A multiresolution analysis consists of a sequence of closed subspaces

$$\dots V_{-2} \subset V_{-1} \subset V_0 \subset V_1 \subset V_2 \dots \quad (2.25)$$

such that it has the following properties (Vetterli and Kovacevic, 1995):

(a) Completeness:

$$\bigcap V_j = \{0\}, \quad \bigcup V_j = L^2(\mathbf{R}) \quad j \in \mathbf{Z} \quad (2.26)$$

(b) Scaling Property:

$$f(t) \in V_j \leftrightarrow f(2t) \in V_{j+1} \quad (2.27)$$

for any function $f \in L^2(\mathbf{R})$

(c) Existence of Basis:

There exists a scaling function $\phi \in V_j$, such that

$$\{\phi_{jk}(t)\} = 2^{j/2} \phi(2^j t - k) \quad \forall j \in \mathbf{Z} \quad (2.28)$$

is an orthonormal basis for V_j .

Equation (2.25) is illustrated via a Venn diagram in Figure 2.5. The basic idea in multiresolution analysis is successive approximations. As can be seen from Figure 2.5, a signal in $L^2(\mathbf{R})$ can be approximated by a coarse representation V_j , which is spanned by ϕ_{jk} . The difference between the original signal and its approximation is the detail. This allows the vector space V_j to be divided into two parts such as

$$V_j = V_{j-1} \oplus W_{j-1} \quad (2.29)$$

$$V_{j-1} \perp W_{j-1},$$

where W_{j-1} is defined as the orthogonal complement of V_{j-1} in V_j , and it provides the detail information to go from subspace V_{j-1} to V_j . The symbol " \oplus " stands for direct sum, and it means that each element of V_j can be written as the sum of the elements of W_{j-1} and the elements of V_{j-1} .

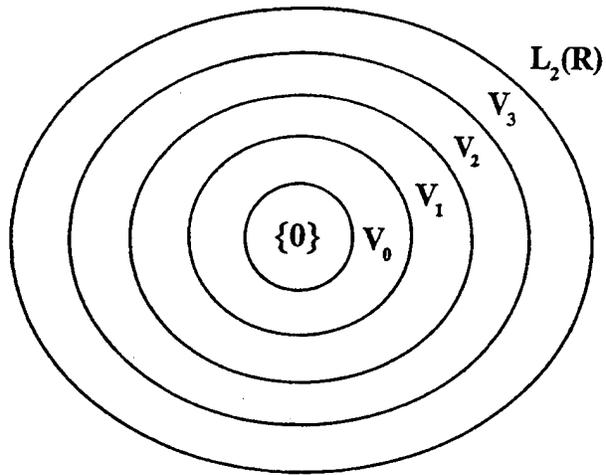


Figure 2.5. Equation 2.25 illustrated via a Venn diagram

$L^2(\mathbf{R})$ also can be represented by the direct sum as

$$L^2(\mathbf{R}) = V_{j_0} \oplus \sum_{j \geq j_0} W_j, \quad (2.30)$$

as illustrated in Figure 2.6.

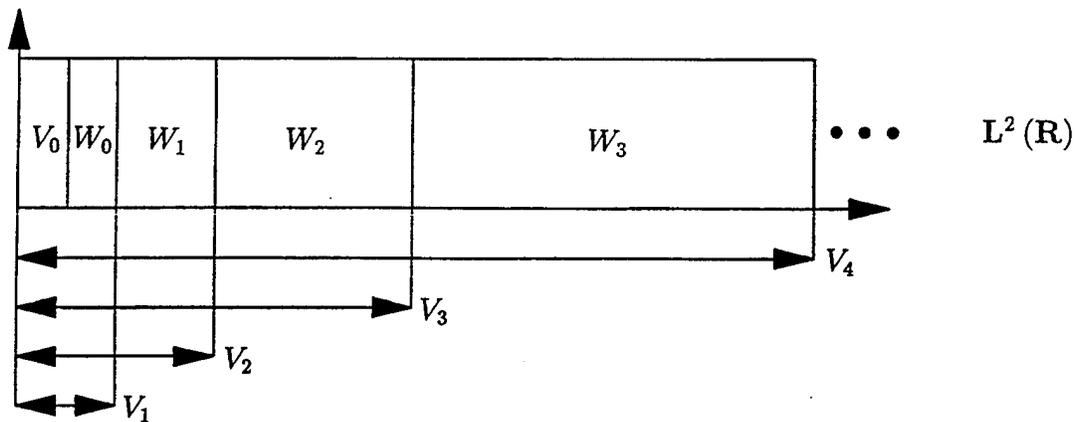


Figure 2.6. Multiresolution representation of $L^2(\mathbf{R})$

b. Scaling and Wavelet Functions

Equation (2.28) gives the scaling function, ϕ_{jk} , which spans the coarse approximation space V_j , and Equation (2.18) gives the wavelet function, ψ_{jk} , which spans the difference information space W_j . Together, they can represent any signal in V_{j+1} . It is also possible to interpret V_j and W_j as lowpass and highpass portions of the space V_{j+1} .

Equations (2.18) and (2.28) imply that both ψ_{jk} and ϕ_{jk} are sets of translates of dilated functions of ψ_j and ϕ_j , respectively. In Equation (2.28), j is a scale factor. When $j > 0$, ϕ_{jk} narrows, \mathbf{V}_j gets wider accordingly, which results in finer detail. If $j < 0$, ϕ_{jk} stretches, \mathbf{V}_j gets narrower, and ends up with coarser resolution.

Let $\phi \in \mathbf{V}_0$ be a scaling function. The translates of the dilated $\phi(t)$ span \mathbf{V}_0 . Thus, the scaling function $\phi(t)$ can be represented as a linear combination of translations of $\phi(2t)$ as

$$\phi(t) = \sum_k h_0(k) \phi(2t - k), \quad k \in \mathbf{Z}, \quad (2.31)$$

where the $h_0(k)$'s form a sequence of real or complex numbers called scaling function coefficients.

Let $\psi(t)$ be an element of subspace \mathbf{W}_0 . The subspace \mathbf{W}_0 itself is a subspace of \mathbf{V}_1 spanned by $\phi(2t)$. Therefore, $\psi(t)$ can be written as a linear combination of translates of $\phi(2t)$:

$$\psi(t) = \sum_k h_1(k) \phi(2t - k), \quad k \in \mathbf{Z}, \quad (2.32)$$

where the $h_1(k)$'s are wavelet coefficients.

It should be noted that since $\phi(t)$ and $\psi(t)$ span two orthogonal spaces, $h_0(k)$ and $h_1(k)$ also should be orthogonal. Hence, orthogonality of \mathbf{V}_0 and \mathbf{W}_0 can be achieved by requiring that

$$\langle h_0(k), h_1(k) \rangle = 0. \quad (2.33)$$

This allows the wavelet coefficients, $h_1(k)$, to be related to the scaling coefficients, $h_0(k)$, such as

$$h_1(k) = (-1)^k h_0(N - 1 - k), \quad (2.34)$$

assuming N is the even number of coefficients. Then, the inner product in Equation (2.33) becomes

$$\begin{aligned}
\langle h_0(k), h_1(k) \rangle &= \sum_{k=0}^{N-1} h_0(k) h_1(k) & (2.35) \\
&= \sum_{k=0}^{N-1} h_0(k) (-1)^k h_0(N-1-k) \\
&= 0.
\end{aligned}$$

Therefore, any function $f(t)$ in $L^2(\mathbf{R})$ can be written as

$$f(t) = \sum_{k=-\infty}^{\infty} a_k \phi_k(t) + \sum_{j=1}^{\infty} \sum_{k=-\infty}^{\infty} d_{jk} \psi_{jk}(t), \quad (2.36)$$

where a_k and d_{jk} are the Discrete Wavelet Transform Coefficients. They are defined as

$$\begin{aligned}
a_k &= \langle f(t), \phi_k(t) \rangle = \int_{-\infty}^{\infty} f(t) \phi_k(t) dt & (2.37) \\
d_{jk} &= \langle f(t), \psi_{jk}(t) \rangle = \int_{-\infty}^{\infty} f(t) \psi_{jk}(t) dt,
\end{aligned}$$

where $\phi_k(t)$ and $\psi_{jk}(t)$ are real functions.

Equation (2.36) states that any function in $L^2(\mathbf{R})$ can be written as a linear combination of the scaling function at a fixed scale plus a linear combination of wavelet functions at higher scales.

The scaling coefficients $h_0(k)$ should be chosen carefully to ensure that the resulting scaling function has compact support. For that reason, they must satisfy some necessary conditions given below (Newland, 1992). The development of these conditions are provided in Appendix A.

(a) The sum of the scaling coefficients should be equal to two.

$$\sum_k h_0(k) = 2. \quad (2.38)$$

(b) If a solution to Equation (2.32) exists, $\int \phi(t) dt = 0$, and $\int |\phi(t)|^2 dt = 1$, and an integer number of translations of $\phi(t)$ are orthogonal to each other, as defined by

$$\langle \phi(t), \phi_k(t) \rangle = \int \phi(t) \phi^*(t-k) dt = \delta(k) = \begin{cases} 1; & k=0 \\ 0; & \text{else} \end{cases}$$

then

$$\sum_n h_0(n) h_0(n - 2k) = 2\delta(k). \quad (2.39)$$

(c) The sum of the squares of the coefficients is equal to two.

Let $k = 0$; Equation (2.39) then becomes

$$\sum_{n=0}^{N-1} |h_0(n)|^2 = 2. \quad (2.40)$$

(d) The individual sums of the odd and even terms of $h_0(k)$ are equal to one.

$$S_1 + S_2 = 2,$$

where N is even and

$$S_1 = \sum_{k=0}^{N/2-1} h_0(2k), \quad (2.41)$$

and

$$S_2 = \sum_{k=0}^{N/2-1} h_0(2k + 1). \quad (2.42)$$

c. Frequency Domain

The DFT of $h_0(k)$ is defined as

$$H(\omega) = \sum_{k=0}^{N-1} h_0(k) e^{-j\omega k}. \quad (2.43)$$

Recall that Equation (2.31) can be interpreted as a convolution between $h_0(k)$ and $\phi(2t)$. In this case, $h_0(k)$ can be viewed as an FIR digital filter impulse response and, accordingly, $H(\omega)$ is the filter's frequency response.

If zero is substituted in frequency ω in Equation (2.43), the d.c. response of the FIR filter is obtained as:

$$H(0) = \sum_k h_0(k). \quad (2.44)$$

The right hand side of Equation (2.44) should be equal to two as the result of the necessary condition stated in Equation (2.38). Thus, the necessary condition, Equation (2.38), is equivalent to requiring the FIR filter's frequency response at d.c. to be two.

If the integer translates of ϕ are orthogonal, then the constraint defined in Equation (2.38) is true, if and only if

$$|H(w)|^2 + |H(w + \pi)|^2 = 4. \quad (2.45)$$

If equation (2.44) is viewed as an FIR filter, $h_0(k)$ is called a quadrature mirror filter impulse response (QMF) (Burrus and Gopinath, 1993).

d. Filter Banks and Discrete Wavelet Transform

Multiresolution analysis can be implemented by using a technique called Multiresolution Pyramid Decomposition or Mallat's algorithm (Vetterli and Kovacevic, 1995).

Let f be a real function in \mathbf{V}_{j+1} . Since $\{2^{\frac{j+1}{2}} \phi(2^{j+1}t - k)\}$ spans \mathbf{V}_{j+1} , f can be represented as

$$f(t) = \sum_k a_{(j+1)k} 2^{\frac{j+1}{2}} \phi(2^{j+1}t - k) = \sum_k a_{(j+1)k} \phi_{(j+1)k}(t), \quad (2.46)$$

where

$$a_{(j+1)k} = \langle f(t), \phi_{(j+1)k}(t) \rangle = \int f(t) 2^{\frac{j+1}{2}} \phi(2^{j+1}t - k) dt. \quad (2.47)$$

Next, since $\mathbf{V}_{j+1} = \mathbf{V}_j \oplus \mathbf{W}_j$, f can be expressed as the sum of two functions, one lying in \mathbf{V}_j and the other in the orthogonal complement \mathbf{W}_j . Hence,

$$f(t) = \sum_k a_{jk} \phi_{jk}(t) + \sum_k d_{jk} \psi_{jk}(t), \quad (2.48)$$

where the DWT coefficients a_{jk} and d_{jk} are given by

$$\begin{aligned} a_{jk} &= \langle \phi_{jk}, f(t) \rangle \\ d_{jk} &= \langle \psi_{jk}, f(t) \rangle. \end{aligned} \quad (2.49)$$

As mentioned earlier, the first term in Equation (2.48) is the coarse approximation, which is the lowpass portion of $f(t)$, whereas the second term contains the detail information, which is the highpass portion of $f(t)$.

From Equations (2.31) and (2.32), we have

$$\begin{aligned} \phi(t) &= \sum_k h_0(k) \phi(2t - k), \\ \psi(t) &= \sum_k h_1(k) \phi(2t - k) \end{aligned}$$

$$\begin{aligned} \phi(2^j t - n) &= \sum_k h_0(k) \phi(2(2^j t - n) - k) \\ &= \sum_k h_0(k) \phi(2^{j+1} t - 2n - k) \end{aligned} \quad (2.50)$$

Let $m = 2n + k$ or $k = m - 2n$, which leads to

$$\phi(2^j t - n) = \sum_m h_0(m - 2n) \phi(2^{j+1} t - m). \quad (2.51)$$

Now, to obtain the DWT coefficients a_{jk} and d_{jk} , Equation (2.50) is substituted into Equation (2.49). Assuming all the values are real,

$$\begin{aligned} a_{jk} &= \langle \phi_{jk}, f(t) \rangle \\ &= \int_{-\infty}^{\infty} f(t) 2^{j/2} \phi(2^j t - k) \\ &= \int_{-\infty}^{\infty} f(t) 2^{j/2} \sum_m h_0(m - 2k) \phi(2^{j+1} t - m) \\ &= \sum_m h_0(m - 2k) 2^{-1/2} \int_{-\infty}^{\infty} f(t) 2^{\frac{j+1}{2}} \phi(2^{j+1} t - m) \\ &= 2^{-1/2} \sum_m h_0(m - 2k) a_{(j+1)m}. \end{aligned} \quad (2.52)$$

$$d_{jk} = 2^{-1/2} \sum_m h_1(m - 2k) a_{(j+1)m}. \quad (2.53)$$

Equations (2.52) and (2.53) are convolutions followed by subsampling. These operations are illustrated in Figure 2.7, where $h_0(k)$ and $h_1(k)$ are given by

$$\begin{aligned} \tilde{h}_0(k) &= h_0(-k) \\ \tilde{h}_1(k) &= h_1(-k). \end{aligned} \quad (2.54)$$

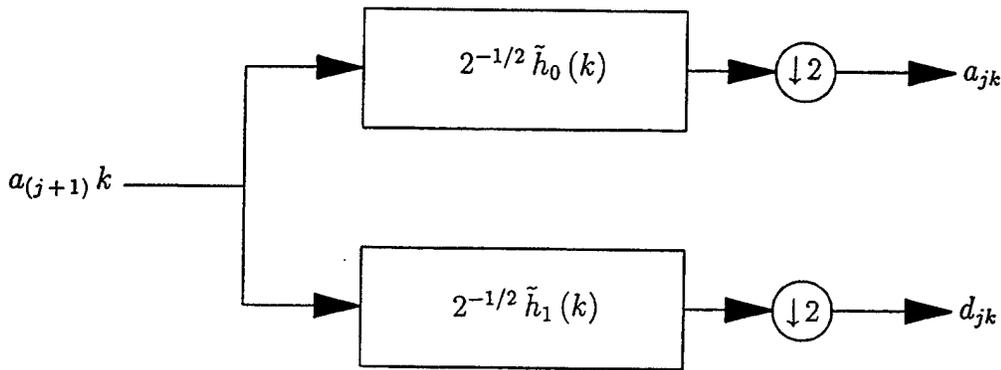


Figure 2.7. One stage of multiresolution signal decomposition

Therefore, the Discrete Wavelet Transform Coefficients at scale j are obtained by convolving the coefficients at scale $j + 1$ with $h_0(-k)$ and $h_1(-k)$, and then decimating to produce the expansion coefficients at scale j . Figure 2.8 shows the implementation of Equations (2.52) and (2.53) for three levels. The notation LP stands for lowpass FIR filter whose weights are $h_0(-k)$, and HP stands for the highpass FIR filter whose weights are $h_1(-k)$.

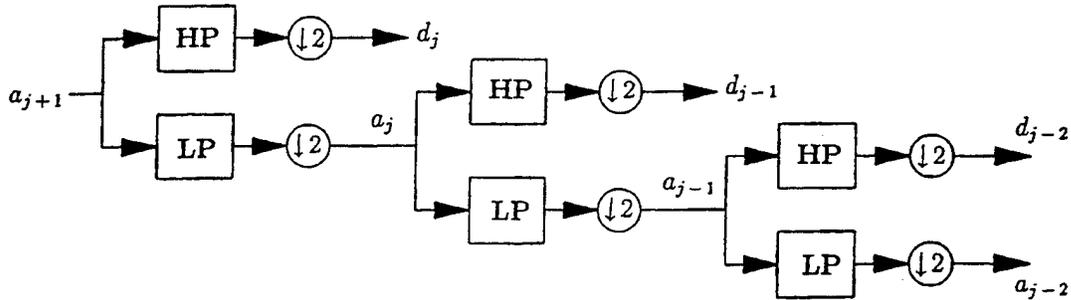


Figure 2.8. Three levels of multiresolution analysis

I. MEDIAN FILTERS

A median filter consists of a sliding window. The center value in the window is replaced by the median of the values within the window. The median of n observations x_i , $i = 1, 2, \dots, n$ is given by

$$\text{med}(x_i) = \begin{cases} x_{(v+1)} & , n = 2v + 1 \\ \frac{1}{2}(x_{(v)} + x_{(v+1)}) & , n = 2v \end{cases} \quad (2.55)$$

where $x_{(i)}$ denotes the center value of n observations (Pitas and Venetsanopoulos, 1990).

A median filter of size $n = 2v + 1$ is defined by the following input-output relation:

$$y_i = \text{med}(x_{i-v}, \dots, x_i, \dots, x_{i+v}) \quad i \in \mathbb{Z},$$

where x_i is the input sequence and y_i is the output sequence. An example of median filtering of size $n = 3$ is shown in Figure 2.9. The input takes on three values: 0, 1, 2. As can be seen in Figure 2.9, the output sequence is 3-valued and does not contain isolated jumps at $n = 2$ and $n = 8$.

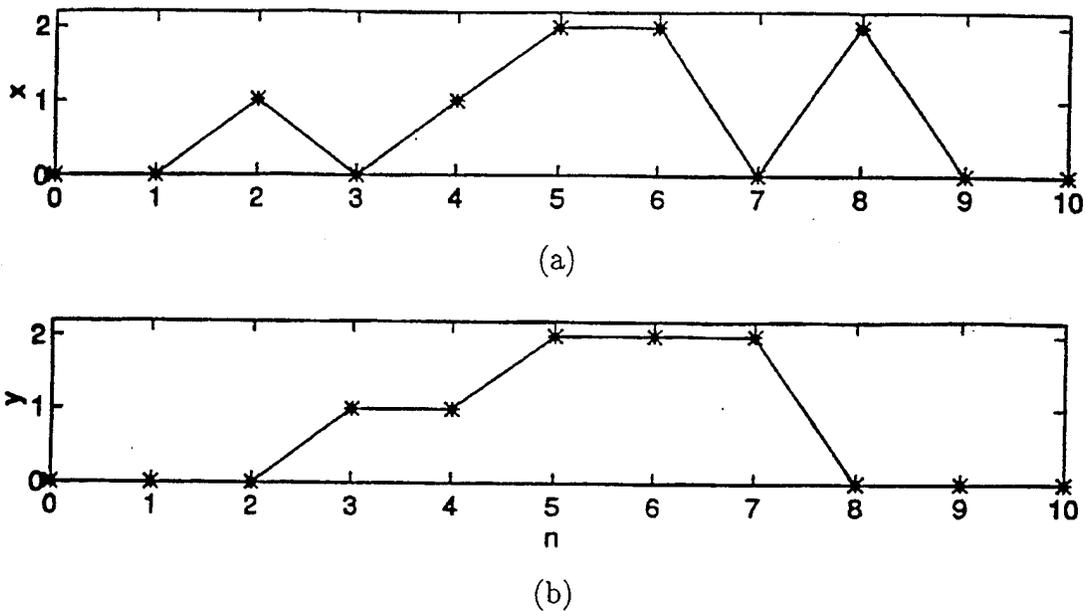


Figure 2.9. (a) Input x to median filter. (b) Output y from median filter.

Median filtering is a nonlinear operation. The main advantage of this kind of nonlinear filtering is its computational simplicity and robustness. Furthermore, it has good edge properties. As will be seen in Chapter III, data processing, wavelet analysis, and signal identification are used to separate the signals according to the transmitter from which they originate. The data processing uses median filtering and extracts the envelope. The wavelet analysis uses a Daubechies wavelet in the Mallat algorithm. The signal identification uses a distance criteria based on selected wavelet scales and wavelet coefficients.

III. WAVELET SIGNAL PROCESSING

A. INTRODUCTION

Mallat's Pyramid Algorithm was used to obtain the Discrete Wavelet Transform (DWT). All programs related to preprocessing and wavelet processing, as well as to signal identification are written in Matlab, a product of Mathworks, Inc. (1992), and are given in Appendix D. The signals used in this analysis do not lend themselves directly to wavelet analysis. Therefore, the processing phase is divided into two parts: a preprocessing phase where the signals are exposed to linear and non-linear operations, and an identification phase, which will be explained in subsequent sections.

The signals are turn-off/turn-on transients from push-to-talk radios. Nine transmissions from four different transmitters were used in the identification part of this thesis.

B. PREPROCESSING PHASE

1. Turn-On Signals from Push-To-Talk Radios

The signals used in the analysis were collected and recorded by the Naval Security Group Activity, Charleston, SC. Nine turn-on/off samples of each of four transmitters were recorded. All the radios were Motorola models. Each radio is identified by its model name and number and is tabulated below:

| Radio: | Model: |
|---------------------|---------|
| Transmitter 1 (Tr1) | Maxtrac |
| Transmitter 2 (Tr2) | Saber |
| Transmitter 3 (Tr3) | HT440 |
| Transmitter 4 (Tr4) | Saber |

Tr2 and Tr4 are different radios of the same model.

The carrier frequency of the radios was 138.525 MHz. The signals were filtered at 1 Mhz Bandwidth (BW), and then digitized with a sampling frequency of 5 Mhz and a center frequency of 1.075 Mhz.

The recordings from each transmitter are numbered from one to nine. Figure 3.1 shows the first turn-on signal of the four transmitters. It can be noticed that

the recordings differ from each other in duration as well as in the type of waveform produced during the transition part. The turn-off response was also examined, but did not lead to any identification clues and was, therefore, not included in the results section.

The information useful for the identification of the transmitter lies in the envelope, which is the low frequency component of the signals. Wavelet Transforms are not useful in analyzing low frequency, but are well-suited for short duration phenomena. Thus, we need to transform the data into a form suitable for wavelet analysis. A preprocessing scheme is applied to the signals to transform them into a usable form, and to improve the signal-to-noise ratio (SNR), as presented in the next section. It should also be noted that the SNR's of the recordings are not known, and a denoising process would definitely enhance the identification performance.

2. Preprocessing Scheme

There are four steps in the preprocessing phase: taking the envelope, median filtering, differentiating, and a final median filtering. Prior to taking the envelopes, the d.c. terms were removed. 100-point median filters were used for filtering the envelope, whereas a 50-point median filter was used after the differentiation. The sizes of the median filters were determined experimentally. The main reason for using median filtering was to avoid broadening the signal, which usually is caused by convolutional operations.

The preprocessed signals for the first recordings from the four transmitters are shown in Figure 3.2. All of the four final pulse-shaped signal waveforms seem to be appropriate for WT analysis. The preprocessing scheme is illustrated step-by-step in Appendix C for the first signal of each transmitter. The final preprocessed pulse-shaped signals for all transmitters are included in Appendix C.

C. IDENTIFICATION PHASE

1. Reduced Set Representation

One of the main drawbacks of the wavelet transform is the shift variance, since the wavelet coefficients of a signal and a shifted replica can be totally different. Therefore, a Euclidean distance measure (introduced by Aware, Inc., 1992) is used as the technique for the classification of signals.

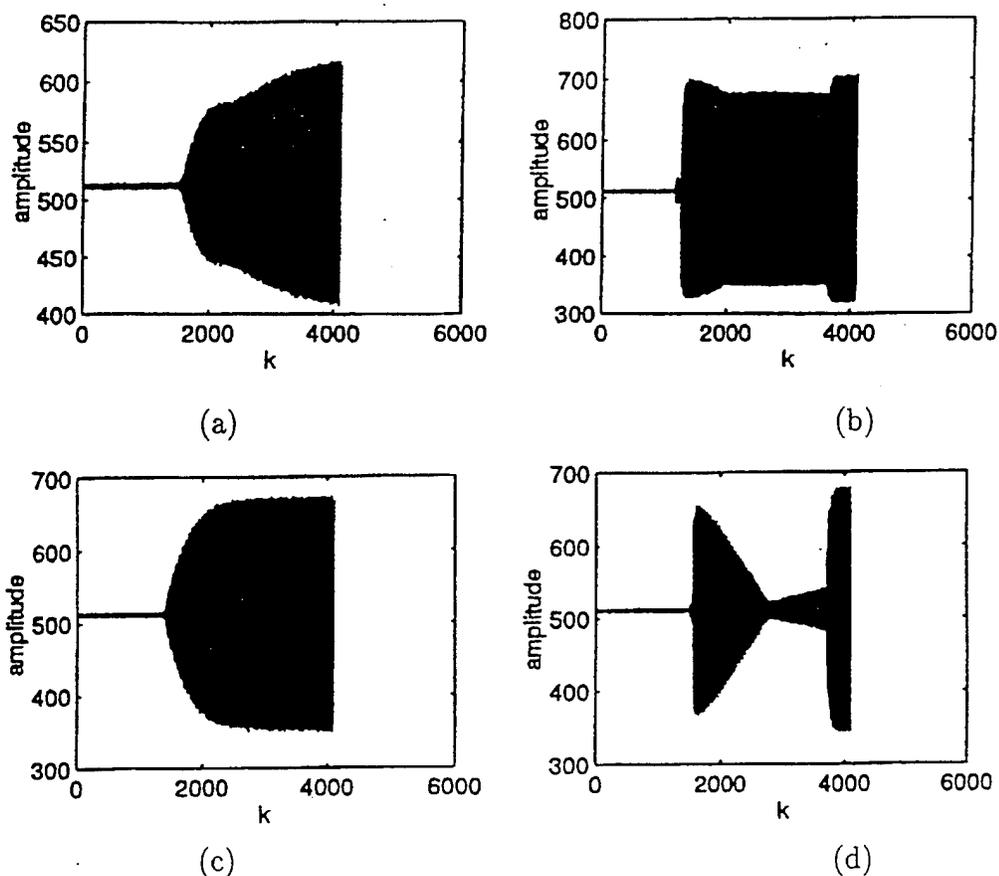


Figure 3.1 Signal samples from four different transmitters

- (a) Signal 1 from Transmitter 1 (b) Signal 1 from Transmitter 2
(c) Signal 1 from Transmitter 3 (c) Signal 1 from Transmitter 4

Mallat and Zhong (1989) demonstrated that the maxima extracted from the modulus of the wavelet coefficients can be used to reconstruct the input signal. That is, the maxima of the modulus of the wavelet coefficients contain approximately the same amount of information as the original signal. Consequently, signal analysis can be performed based on the wavelet extremals, which form a reduced signal representation.

Thus, wavelet coefficients at each scale have been replaced by their extremal values. Hence, the reduced set is only nonzero where the scale has an extremal, and is equal to the original value at these locations. Wavelet scale coefficients, which are not extremals, are set to zero.

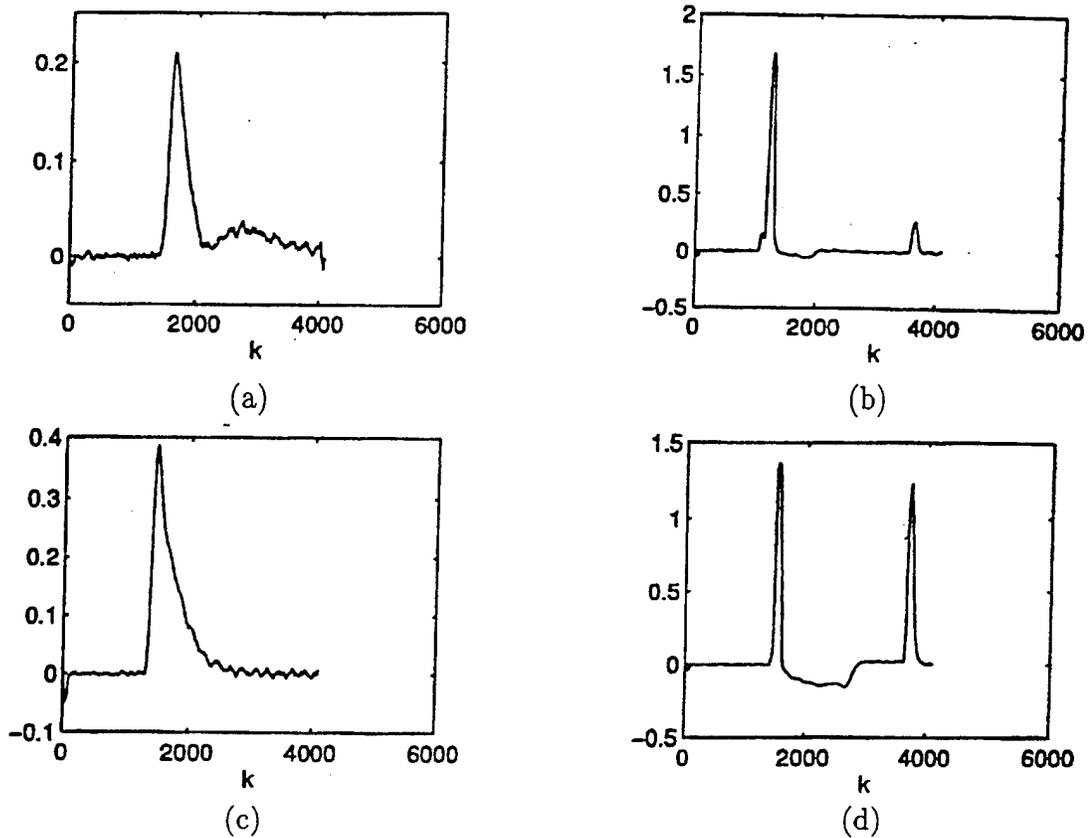


Figure 3.2 Preprocessed signal samples from four different transmitters

- | | |
|---------------------------------|---------------------------------|
| (a) Signal 1 from Transmitter 1 | (b) Signal 1 from Transmitter 2 |
| (c) Signal 1 from Transmitter 3 | (d) Signal 1 from Transmitter 4 |

2. Ranking/Matching Algorithm

The first step in computing a pairwise distance measure is to rank the peaks of the sets to be compared. That is, the peaks at each scale are ranked by their amplitudes. The second step is to match the ranked peaks to form pairs. The peak with the highest rank in one set is matched to the peak with the highest rank in the other set. The next in rank is matched to that of the other set, and so on. To illustrate the ranking and matching scheme, let us consider $f(k)$ and $g(k)$, as shown in Figure 3.3. We first rank the peaks by amplitude. The amplitudes of the signals in ascending order are -2, -1, 1, 2, 3, 5 for $f(k)$, and -1, 0, 0.5, 1, 2.5, 4 for $g(k)$. Then, we match the peaks by taking corresponding ranked peaks and forming pairs. The peak with the highest rank in $f(k)$ is 5; the peak with the highest rank in

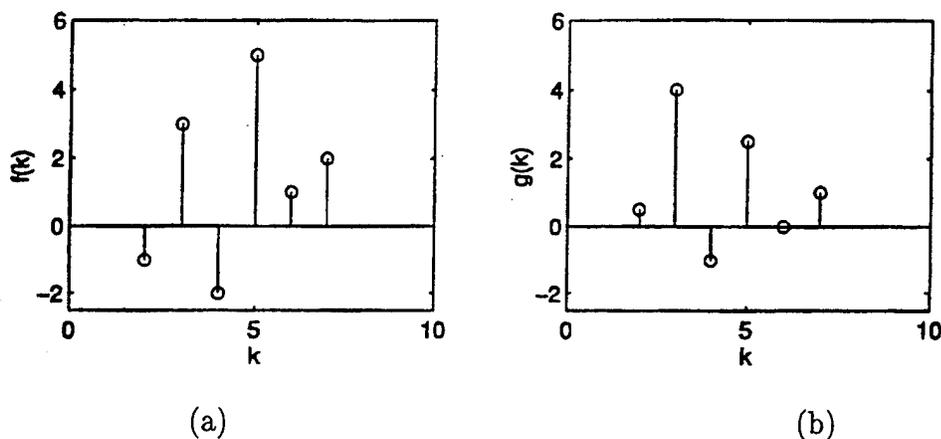


Figure 3.3 Signals to be matched:
 (a) maxima of signal $f(k)$ (b) maxima of signal $g(k)$

| | | | | | |
|------------|--------------|------------|--------------|-------------|-------------|
| $f(5) = 5$ | $f(3) = 3$ | $f(7) = 2$ | $f(6) = 1$ | $f(1) = -1$ | $f(4) = -2$ |
| $g(3) = 4$ | $g(5) = 2.5$ | $g(7) = 1$ | $g(2) = 0.5$ | $g(6) = 0$ | $g(4) = -1$ |

Table 3.1 Matched pairs of signals $f(k)$ and $g(k)$

(columns show matched pairs, ranked in decreasing order from left to right)

$g(k)$ is 4. We match these peaks with highest ranks first. Then, we match the next in rank, which are 3 from $f(k)$ and 2.5 from $g(k)$, and so on. The ranked and matched pairs are tabulated in Table 3.1. The matching scheme does not require the number of elements in both sets to be the same. When a peak in one set does not have a corresponding peak in the other, we insert a zero into the set that has a smaller number of peaks, and form the pairs by matching the remaining peaks and the zeros.

3. Distance Measure

The third step is to compute a distance measure for the matched pairs. The distance assigned to the pair is the sum over the Euclidean distances in each scale. Thus,

$$d(a^j, b^j) = \sum_{(k,l) \text{ are locations of the matched peaks}} [W_{k,l}^j (a_k^j - b_l^j)^2]^{\frac{1}{2}} \quad (3.1)$$

where a^j and b^j are the wavelet maxima at scale j , and a_k^j and b_l^j are the values a^j , b^j at temporal locations k and l , respectively, and $W_{k,l}^j$ is the weighting factor at scale j for the relative distances between the corresponding coordinates of matched peaks. The weighting factor, $W_{k,l}^j$, is defined as

$$W_{k,l}^j = |n_k^j - n_l^j| \quad (3.2)$$

where n_k^j and n_l^j are the coordinates of a_k^j and b_l^j , respectively.

The similarity between two sets is described in terms of the sum of the Euclidean distances of amplitudes weighted by the square root of the relative distances between the corresponding coordinates. Basically, a small value $|n_k^j - n_l^j|$ and $|a_k^j - b_l^j|$ imply a high degree of similarity between a^j and b^j .

As can be seen from Equation (3.1), this method provides no penalty if the peaks happen to be at the same location, or the amplitudes of the peaks are equal to each other. The penalty weight, $W_{k,l}^j$, is determined by the separation of the peaks. Large separation between the matched peaks corresponds to large penalty factors. The distance measure also depends on the amplitude difference of the matched peaks. The distance measure is directly proportional to the difference in amplitude. It should be noted that even identical scale outputs with offset in time (due to signal delay) will have a nonzero weighting factor.

4. Modification to Weighting Factor

We now introduce a lineup procedure and a modified weighting term to offset the shortcoming of the technique, described in Section III.C.3. This is the most robust method for obtaining reliable distance measures, and is used to obtain the experimental results.

There are four different sets of signals. Each set belongs to a different transmitter. The method defined above was used to determine the transmitting source of the signal. For this purpose, a template for each transmitter set is needed to measure the similarity with any signal of interest. The objective is to apply the distance measure algorithm to identify the transmitters.

Matching peaks by ranking preserves the order of the peaks when the signals to be compared are very similar (i.e., identical), which then causes the weighting factor in Equation (3.2) to be a constant denoted by W . For example, if we shift $f(k)$ in

Figure 3.3 four units to the right and compare it to $f(k)$, the weighting factor in \mathbf{W} in Equation (3.2) is 4 for all matched peaks. Since the amplitudes of the matched peaks are the same, the distance measure defined in Equation (3.1) is zero, in spite of the nonzero weighting factor \mathbf{W} . This is not the case when we add noise to one or both of the signals, because it will cause amplitude difference between matched peaks. Even though the signals are similar, a nonzero distance measure will be computed. Furthermore, this distance measure will be dominated by the constant weighting factor \mathbf{W} . We need to reduce the weighting factor in such cases by eliminating the offset between the signals. To eliminate this problem, the maximum peaks of the two sets are aligned by removing the time shift offset between the signals.

The first signal of the four sets was chosen as a template. To improve the accuracy of the algorithm, every signal was aligned with each template before applying the distance measure, Equation (3.1). Hence, the relative distance between the maximum matched peaks of the signal and templates is zero. Accordingly, the weighting factor, $\mathbf{W}_{k,l}^j$, will be zero for the maximum peaks for all templates. Applying zero weight for the distance with the corresponding (i.e., proper) template makes sense, but not for the other templates. Therefore, the weighting factor is modified to be

$$\mathbf{W}_{k,l}^j = \begin{cases} |n_k^j - n_l^j| & k \neq l \\ 1 & k = l. \end{cases} \quad (3.3)$$

Now, a small distance measure implies more accurately that the signal belongs to the set from which the template came.

5. Application of Distance Measure Algorithm

The fourth, eighth, and sixteenth order Daubechies wavelet functions were used to compute the WT of the preprocessed signals. The distance measure algorithm was applied to measure the similarity between the wavelet coefficients of the signal to be identified and the templates. From experimental considerations, the eighth order Daubechies (Dau-8) wavelet functions gave satisfactory results, whereas Dau-4 and Dau-16 did not. The Dau-8 wavelet function, which was used as the mother wavelet, is shown in Figure 3.4.

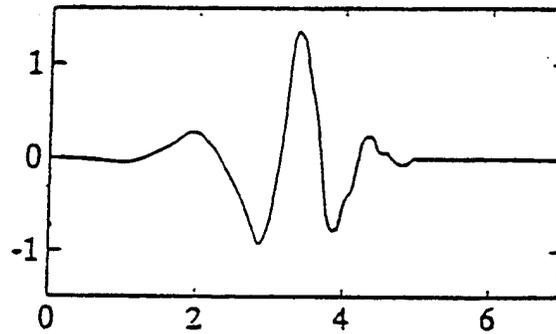


Figure 3.4 Eighth order Daubechies wavelet function

a. Transmitter 1

The distance values at Scale 11 between the first four signals from Transmitter 1 and the templates are tabulated and shown in Table 3.2. As expected, the distance value with Template 1 is significantly less than that with the other templates, since Template 1 is the template for the signals from Transmitter 1, and small distance values imply high similarity.

| <i>Scale 11</i> | Template 1 | Template 2 | Template 3 | Template 4 |
|-----------------|------------|------------|------------|------------|
| Signal 1 | 0 | 44.3073 | 14.7737 | 107.8438 |
| Signal 2 | 3.0527 | 47.2375 | 16.3406 | 103.8587 |
| Signal 3 | 2.6195 | 40.5894 | 13.797 | 99.7881 |
| Signal 4 | 1.0806 | 42.9072 | 15.4847 | 105.308 |

Table 3.2 Distance measures at Scale 11 for Signals 1 – 4 of Transmitter 1

As can be seen from Table 3.2, the distance values, d , between signals from Transmitter 1 and Template 1 are well separated from those of the other templates. Zero value in distance indicates perfect match, and occurs between Signal 1 and Template 1. This is expected since Signal 1 was chosen as a template for the set.

A thresholding technique can be introduced to automate the identification procedure by defining a threshold level for each template. For example, one can compare the maximum value of Set 1 with the minimum values of other templates. Determining the threshold levels is not addressed in this study; however, comparing the minimum and maximum values of the template outputs may be quite useful to

evaluate the performance of the identification scheme. The maximum and minimum values quoted in the remainder of Chapter 3 are obtained from Appendix B. At Scale 11 with Template 1, the maximum distance value is 5.3971, whereas the nearest minimum distance value of the other templates is 12.52. This minimum occurs with Template 3, which indicates that Transmitter 1 and Transmitter 3 have a somewhat similar output (i.e., turn-on behavior). The ratio of the minimum Template 3 output to the maximum Template 1 output is 2.319, which shows how well the Template 1 results are separated from the other template results. This separation ratio is 2.381 and 1.66 for Scale 10 and Scale 9, respectively. Lower scales were not useful in terms of the similarity measurement, and were not included.

b. Transmitters 2, 3, and 4

Similarly, the application of the distance measure to the other three sets resulted in the following separation ratios. They show how well the signals of a particular set are separated from the other sets under worst case conditions. They are 3.56, 2.76, and 1.88 for signals from Transmitter 2 at Scales 11, 10, and 9, respectively. The following separation ratios were obtained at Scales 11, 10, and 9: 4.64, 1.98, and 3.73 for signals from Transmitter 3, and 1.75, 1.4, 1.24 for signals from Transmitter 4. Signal versus template output plots for all signal sets are given in Appendix C. For example, one can obtain minima, maxima, and standard deviation of the distance measures very easily. It should be noted that, for Transmitters 1, 2, and 3, nine signals were used while, for Transmitter 4, only six signals were usable.

c. Probability of Identification

Signal-to-Noise Ratios (SNRs) for all signals were estimated. As seen in Figure (3.1), the signal waveforms can be split into three parts. The off region is where there is no signal; the transition region is where there is a build-up from off to on state; and the on region is where the signal is at steady state. We can assume that the off region consists of noise only, allowing the computation of the noise power. Noise and signal coexist in the on region. The signal power can be computed by finding the power in the on region and subtracting the noise power. Hence, we can compute the SNR's for all sets. An average SNR value of 32.38 dB, 40.87 dB, 38.58 dB, and 31 dB was computed for Set 1, Set 2, Set 3, and Set 4, respectively.

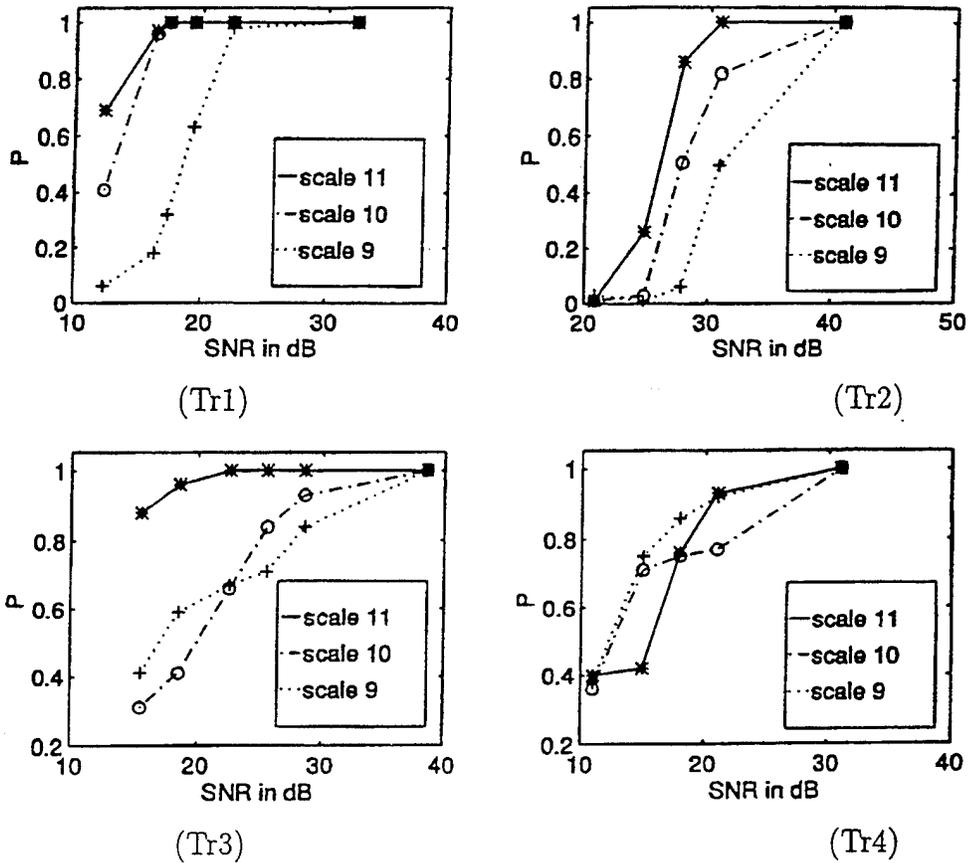


Figure 3.5 Probability of identification (P_i) of the signals

- (a) P_i of signals from Transmitter 1 (b) P_i of signals from Transmitter 2
(c) P_i of signals from Transmitter 3 (c) P_i of signals from Transmitter 4

Gaussian white noise was added to all sets to decrease the SNR levels. We repeated the experiment eleven times for different SNR values, and obtained the probability of identification versus SNR plots for the signals. Results are shown in Figure 3.5. Note that, when the signal of interest is from Transmitter 1 and its SNR is higher than 16 dB, then its identification probability is high, and the distance-based algorithm is reliable. The minimum SNR values required for a reliable classification at Scale 11 are 30, 23, and 30 dB for signals from Transmitter 2, 3, and 4, respectively.

d. Attempts to Reduce the Processing Load

Using a reduced data set (i.e., taking the local extremal in distance computation) reduced the computational complexity. In addition, two approaches were attempted to obtain acceptable performance while minimizing the computational effort. In the first approach, the signal was decimated by using a Chebychev -2 filter after median filtering of the envelope. This reduced the number of points and, therefore, the number of computations. The distance algorithm using the decimated data did not work well enough to classify the signals and was not pursued further. In the second approach, instead of matching all the maxima of the signal of interest and the template, only the highest four peaks from each were matched, and a distance measure was computed. Even the original data did not permit successful classification; hence, noisy versions of the signal were not tried.

Matching of the peaks of the signal and template includes ranking. Therefore, it is highly affected by noise. To minimize the noise effect in the matching of peaks, point-by-point matching was used instead of ranking, but the results were not satisfactory. In summary, distance measures that use all extremals and match them by ranking are the most effective scheme in classifying the four signal sets.

IV. CONCLUSION

A. CLASSIFICATION OF PUSH-TO-TALK COMMUNICATION SIGNALS

The main objective of this study was to determine a wavelet-based algorithm designed to extract features to identify push-to-talk transmitters. Robustness, compact signal representation capability, and low computational complexity are the main advantages of the wavelet analysis, which was used for feature extraction.

Push-to-talk communication recordings provided for this research have a common feature: They all include a transition from the off-to-on state, as well as the on-to-off state. The turn-on transition phase is unique for each transmitter, and can be used for classification purposes. The recordings differ from each other by the waveform in the transition region and the duration. The feature for the classification of the signals is contained in the signal envelopes. The turn-on part of the envelope is a transient and, hence, a wideband signal.

The push-to-talk recordings were not in appropriate forms for WT analysis. Thus, the recordings were preprocessed to become usable for WT processing. Differentiation of the envelope of the signals was used to transform the data into pulse-shaped transients. Median filtering was applied to both the envelope and the differential to improve the signal-to-noise ratio. Median filtering was preferred since it does not broaden the transient features of the signal.

A distance algorithm was introduced in this work. It was based on a Euclidean distance measure between the wavelet coefficients of two data set on a given scale. Decisions about the origin of the signal was made according to the distance measures between the signals and the templates, where each template represented a different transmitter. A small distance value implied that the signal belonged to the same set as that particular template. In its current form, the classification assumes that any signal of interest belongs to one of the four sets.

The distance algorithm was applied to four different signal sets. The first recording in any of the sets was designated to be the template. Instead of using all the wavelet coefficients, just the local extrema were used. Using only edge points by taking the local extrema reduced the computational complexity in the algorithm.

The distance measure between the signal and the templates is the sum of all Euclidean distances between matched peaks of the signal and the templates. It also includes a penalty factor due to the relative square root distance between the matched ranked pairs and their difference in amplitude. Matching signal peaks to the template is performed by ranking. Before matching the pairs, the maximum peak of the signal is aligned with the maximum of the templates. This reduced the penalty weight for like signals that are not aligned in time.

The WT of the signals was computed using Daubechies-4 (Dau-4), Daubechies-8 (Dau-8), and Daubechies-16 (Dau-16) as mother wavelets. Experimentally, it was found that Dau-8 worked best with this data set.

Decimating the signal envelope before taking the differential reduces the number of points and, hence, decreases the number of computations. The distance algorithm was applied to the decimated sequences, but the results did not allow classification of the signals.

We also considered using only the four highest wavelet coefficients at a given scale in the distance computation to reduce the computational complexity. However, simulations showed that the information cited was not sufficient. The distance algorithm described in Section III, using all the extremas, matching the peaks by ranking, and permitting a penalty according to Equation (3.3) allowed robust identification of the signal sets.

B. RECOMMENDATION FOR FUTURE STUDIES

The distance algorithm introduced in Chapter III gave promising results in classifying the four signal sets provided for this research. Three important issues have not been addressed in this research: the template selection, a threshold technique, and incorporation of information from other scales. In this work, templates were chosen arbitrarily from the sets, and the distance algorithm was applied only to the four signal sets. When the signals to be identified are from these sets, the algorithm was capable of classifying the signals. However, if the signals do not belong to these sets, the algorithm will compute a distance to each of the templates, which can lead to misclassification. A thresholding technique and robust template selection should be the subject of further study.

Also, no attempt was made to combine information from several scales. Identification was obtained by using just one scale. Typically, Scale 11 worked best. If the distance information from Scales 9 and 10 could be used, a more robust identification performance should be realized.

APPENDIX A. DEVELOPMENT OF NECESSARY CONDITIONS

NECESSARY CONDITION 1:

The condition given by Equation (2.38) can be derived by integrating both sides of Equation (2.31) (Burrus and Gopinath, 1993).

$$\begin{aligned} \int \phi(t)dt &= \int \sum_k h_0(k)\phi(2t-k)dt & (A.1) \\ &= \sum_k h_0(k) \int \phi(2t-k)dt. \end{aligned}$$

Let $m = 2t - k$, so $dt = \frac{1}{2}dm$. Substituting into Equation (A.1) gives

$$\begin{aligned} \int \phi(t)dt &= \frac{1}{2} \sum_k h_0(k) \int \phi(m)dm, & (A.2) \\ \frac{\int \phi(t)dt}{\int \phi(m)dm} &= \frac{1}{2} \sum_k h_0(k). \end{aligned}$$

The left side of Equation (A.2) is equal to one, and so

$$\frac{1}{2} \sum_k h_0(k) = 1,$$

or

$$\sum_k h_0(k) = 2. \quad (A.3)$$

NECESSARY CONDITION 2

ϕ_k forms an orthonormal set for $L^2(\mathbb{R})$. Therefore, the inner product of $\phi(t)$ with integer translate of itself is

$$\langle \phi(t), \phi_k(t) \rangle = \int \phi(t)\phi(t-k) = \delta(k). \quad (A.4)$$

Substituting Equation (2.31) into (A.4) gives

$$\begin{aligned} \int \sum_n \sum_m h_0(n)h_0(m)\phi(2t-n)\phi(2(t-k)-m)dt &= \delta(k), \\ \sum_n \sum_m h_0(n)h_0(m) \int \phi(2t-n)\phi(2(t-k)-m)dt &= \delta(k). \end{aligned}$$

Let $2t = \tau$, so $dt = \frac{d\tau}{2}$ and

$$\sum_n \sum_m h_0(n)h_0(m) \frac{1}{2} \int \phi(\tau - n)\phi(\tau - 2k - m)d\tau = \delta(k).$$

Since the integral is equal to $\delta(n - m - 2k)$ from (A.4), then

$$\frac{1}{2} \sum_n \sum_m h_0(n)h_0(m)\delta(n - m - 2k) = \delta(k). \quad (\text{A.5})$$

Let $n = m + 2k$; then Equation (A.5) becomes

$$\sum_n h_0(n)h_0(n - 2k) = 2\delta(k). \quad (\text{A.6})$$

NECESSARY CONDITION 3

Let $k = 0$; then Equation (A.6) becomes

$$\begin{aligned} \sum_n h_0(n)h_0(n) &= 2, \\ \sum_n |h_0(n)|^2 &= 2. \end{aligned} \quad (\text{A.7})$$

NECESSARY CONDITION 4

Reproducing Equation (A.6)

$$\sum_n h_0(n)h_0(n - 2k) = 2\delta(k).$$

Summing over k of both sides gives

$$\sum_k \sum_n h_0(n)h_0(n - 2k) = 2. \quad (\text{A.8})$$

Breaking Equation (A.8) into even and odd ordered terms, we obtain

$$\sum_k \left[\sum_m h_0(2m)h_0(2k + 2m) + \sum_m h_0(2k + 2m + 1)h_0(2m + 1) \right] = 2 \quad (\text{A.9})$$

Rearranging terms results in

$$\sum_m \left[\sum_k h_0(2k + 2m) \right] h_0(2m) + \sum_m \left[\sum_k h_0(2k + 2m + 1) \right] h_0(2m + 1) = 2. \quad (\text{A.10})$$

Substituting Equations (2.41) and (2.42) into Equation (A.10), gives

$$S_1 \sum_m h_0(2m) + S_2 \sum_m h_0(2m+1) = 2. \quad (\text{A.11})$$

Applying the same equations again into Equation (A.11) gives

$$S_1^2 + S_2^2 = 2. \quad (\text{A.12})$$

Equations 2.41 and 2.42 state that

$$S_1 + S_2 = 2. \quad (\text{A.13})$$

Solving for S_1 and S_2 in Equations (A.11) and (A.12), gives

$$S_1 = \sum_n h_0(2n) = 1, \quad (\text{A.14})$$

$$S_2 = \sum_n h_0(2n+1) = 1. \quad (\text{A.15})$$

APPENDIX B. DISTANCE MEASURES

A. SIGNALS FROM TRANSMITTER 1 ARE THE INPUTS

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| SIGNAL 2: | 0.0154 | 0.0189 | 0.1321 | 0.1166 | 0.5972 | 3.0527 |
| SIGNAL 3: | 0.0119 | 0.0357 | 0.0512 | 0.1026 | 0.6823 | 2.6195 |
| SIGNAL 4: | 0.0119 | 0.0090 | 0.0518 | 0.1234 | 0.3941 | 1.0806 |
| SIGNAL 5: | 0.0096 | 0.0596 | 0.0811 | 0.1272 | 1.1857 | 4.2015 |
| SIGNAL 6: | 0.0187 | 0.0144 | 0.1260 | 0.1907 | 0.5007 | 2.1553 |
| SIGNAL 7: | 0.0088 | 0.0156 | 0.0644 | 0.2489 | 0.5075 | 3.4189 |
| SIGNAL 8: | 0.0184 | 0.0353 | 0.0335 | 0.1883 | 0.5085 | 2.7106 |
| SIGNAL 9: | 0.0108 | 0.0349 | 0.1020 | 0.4137 | 1.0445 | 5.3966 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.2835 | 0.2481 | 0.5950 | 2.5081 | 9.6966 | 44.3073 |
| SIGNAL 2: | 0.3811 | 0.2786 | 0.6446 | 2.4572 | 8.9884 | 47.2375 |
| SIGNAL 3: | 0.3828 | 0.3385 | 0.4982 | 2.3074 | 8.6315 | 40.5894 |
| SIGNAL 4: | 0.2978 | 0.2723 | 0.5886 | 2.3877 | 8.8995 | 42.9072 |
| SIGNAL 5: | 0.3043 | 0.3478 | 0.4959 | 2.3277 | 8.1022 | 39.6375 |
| SIGNAL 6: | 0.3253 | 0.2469 | 0.8060 | 2.3600 | 9.1666 | 43.6621 |
| SIGNAL 7: | 0.2867 | 0.2623 | 0.5220 | 2.0454 | 9.1751 | 39.6036 |
| SIGNAL 8: | 0.3772 | 0.3061 | 0.5358 | 2.1604 | 8.8586 | 40.6839 |
| SIGNAL 9: | 0.3756 | 0.3014 | 0.5133 | 1.9071 | 8.6736 | 39.2300 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0293 | 0.0478 | 0.1806 | 0.7911 | 3.2748 | 14.7737 |
| SIGNAL 2: | 0.0195 | 0.0516 | 0.2315 | 0.7833 | 3.4095 | 16.3406 |
| SIGNAL 3: | 0.0292 | 0.0873 | 0.1219 | 0.7441 | 3.3865 | 13.7970 |
| SIGNAL 4: | 0.0147 | 0.0461 | 0.1442 | 0.7797 | 3.1756 | 15.4847 |
| SIGNAL 5: | 0.0247 | 0.1011 | 0.1338 | 0.7385 | 2.8243 | 13.1612 |
| SIGNAL 6: | 0.0143 | 0.0448 | 0.1829 | 0.6879 | 3.2091 | 13.4287 |
| SIGNAL 7: | 0.0215 | 0.0571 | 0.1285 | 0.8024 | 3.2493 | 13.7654 |
| SIGNAL 8: | 0.0454 | 0.0721 | 0.1630 | 0.7261 | 3.3426 | 14.3787 |
| SIGNAL 9: | 0.0333 | 0.0683 | 0.1201 | 0.7172 | 3.0344 | 12.5220 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3094 | 0.5771 | 1.2600 | 4.7861 | 23.4442 | 107.8438 |
| SIGNAL 2: | 0.3395 | 0.5918 | 1.3712 | 4.9459 | 21.3511 | 103.8587 |
| SIGNAL 3: | 0.3230 | 0.6083 | 1.2067 | 4.6558 | 21.4529 | 99.7881 |
| SIGNAL 4: | 0.3675 | 0.5963 | 1.2665 | 4.8759 | 22.1737 | 105.3080 |
| SIGNAL 4: | 0.3766 | 0.6255 | 1.1815 | 4.7152 | 20.7366 | 101.1619 |
| SIGNAL 6: | 0.3035 | 0.5860 | 1.3969 | 4.9315 | 22.9453 | 110.3185 |
| SIGNAL 7: | 0.3733 | 0.6054 | 1.1867 | 4.3094 | 22.0092 | 95.2878 |
| SIGNAL 8: | 0.2758 | 0.5979 | 1.2183 | 4.4715 | 21.8996 | 99.7182 |
| SIGNAL 9: | 0.3427 | 0.6121 | 1.1942 | 4.3671 | 21.8617 | 101.6282 |

B. SIGNALS FROM TRANSMITTER 2 ARE THE INPUTS

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.2835 | 0.2481 | 0.5950 | 2.5081 | 9.6966 | 44.3073 |
| SIGNAL 2: | 0.1008 | 0.2111 | 0.5449 | 2.4071 | 9.9080 | 42.9127 |
| SIGNAL 3: | 0.2231 | 0.2929 | 0.6255 | 2.2173 | 9.5509 | 45.2351 |
| SIGNAL 4: | 0.1587 | 0.3089 | 0.7071 | 2.4259 | 10.2745 | 45.9598 |
| SIGNAL 5: | 0.2397 | 0.3638 | 0.6821 | 2.6701 | 10.6592 | 50.7797 |
| SIGNAL 6: | 0.1881 | 0.2625 | 0.6220 | 2.6232 | 10.7310 | 49.5033 |
| SIGNAL 7: | 0.1906 | 0.3823 | 0.6591 | 2.3317 | 10.0235 | 50.3606 |
| SIGNAL 8: | 0.3259 | 0.2937 | 0.7425 | 2.5765 | 11.2478 | 49.7514 |
| SIGNAL 9: | 0.3105 | 0.3787 | 0.6678 | 2.5760 | 11.1999 | 52.8772 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| SIGNAL 2: | 0.1504 | 0.1834 | 0.2414 | 1.1648 | 2.9675 | 8.3316 |
| SIGNAL 3: | 0.0926 | 0.1289 | 0.2083 | 0.3845 | 2.0463 | 11.7426 |
| SIGNAL 4: | 0.1542 | 0.1445 | 0.2714 | 0.2385 | 2.9824 | 10.2347 |
| SIGNAL 5: | 0.1100 | 0.1513 | 0.1427 | 0.2236 | 1.4235 | 12.0260 |
| SIGNAL 6: | 0.0919 | 0.0599 | 0.0953 | 0.2324 | 3.0988 | 3.5685 |
| SIGNAL 7: | 0.1120 | 0.0864 | 0.2697 | 1.0350 | 0.4110 | 6.5288 |
| SIGNAL 8: | 0.0429 | 0.0602 | 0.3798 | 0.2762 | 3.4554 | 2.7844 |
| SIGNAL 9: | 0.0585 | 0.1117 | 0.2090 | 1.1790 | 3.0197 | 8.5858 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3185 | 0.2155 | 0.5388 | 2.5937 | 10.7275 | 52.0090 |
| SIGNAL 2: | 0.1653 | 0.2228 | 0.4821 | 2.8090 | 10.7830 | 46.1492 |
| SIGNAL 3: | 0.3053 | 0.3015 | 0.6483 | 2.4348 | 10.0656 | 48.1584 |
| SIGNAL 4: | 0.2162 | 0.3174 | 0.7455 | 2.4423 | 10.9963 | 48.9938 |
| SIGNAL 5: | 0.3320 | 0.3800 | 0.6067 | 2.6860 | 11.4619 | 55.0565 |
| SIGNAL 6: | 0.2723 | 0.2287 | 0.6431 | 2.5235 | 11.5436 | 55.4853 |
| SIGNAL 7: | 0.2483 | 0.3425 | 0.7296 | 2.5858 | 10.9103 | 55.0225 |
| SIGNAL 8: | 0.3857 | 0.2846 | 0.8665 | 2.5850 | 12.4298 | 54.1137 |
| SIGNAL 9: | 0.4016 | 0.3610 | 0.6024 | 2.8528 | 11.8614 | 57.5092 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0688 | 0.3144 | 0.7227 | 2.4151 | 10.7828 | 46.0266 |
| SIGNAL 2: | 0.2709 | 0.4742 | 0.7804 | 2.4630 | 12.0574 | 53.2623 |
| SIGNAL 3: | 0.1439 | 0.2997 | 0.7299 | 2.3018 | 12.1275 | 51.6991 |
| SIGNAL 4: | 0.2207 | 0.3734 | 0.7884 | 2.6920 | 12.1243 | 49.1001 |
| SIGNAL 5: | 0.1785 | 0.2800 | 0.6635 | 2.7456 | 11.6775 | 49.5180 |
| SIGNAL 6: | 0.1251 | 0.3715 | 0.8402 | 2.9717 | 12.1831 | 47.9240 |
| SIGNAL 7: | 0.1568 | 0.2853 | 0.8519 | 2.6483 | 11.8194 | 52.1264 |
| SIGNAL 8: | 0.1382 | 0.2722 | 0.8705 | 2.6883 | 11.7220 | 47.9139 |
| SIGNAL 9: | 0.1242 | 0.3436 | 0.6856 | 2.4425 | 12.4913 | 50.0139 |

C. SIGNALS FROM TRANSMITTER 3 ARE THE INPUTS

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0293 | 0.0478 | 0.1806 | 0.7911 | 3.2748 | 14.7737 |
| SIGNAL 2: | 0.0258 | 0.0486 | 0.1886 | 0.7386 | 3.4439 | 16.0413 |
| SIGNAL 3: | 0.0258 | 0.0488 | 0.1760 | 0.7199 | 2.9349 | 14.0412 |
| SIGNAL 4: | 0.0243 | 0.0380 | 0.2365 | 0.8262 | 3.1571 | 15.9305 |
| SIGNAL 5: | 0.0351 | 0.0413 | 0.2032 | 0.7480 | 3.3249 | 15.7214 |
| SIGNAL 6: | 0.0181 | 0.0522 | 0.1437 | 0.6904 | 3.3154 | 15.6880 |
| SIGNAL 7: | 0.0204 | 0.0407 | 0.1965 | 0.6858 | 3.0757 | 14.9835 |
| SIGNAL 8: | 0.0211 | 0.0487 | 0.1396 | 0.7145 | 3.1253 | 14.8495 |
| SIGNAL 9: | 0.0132 | 0.0385 | 0.1556 | 0.6820 | 2.6568 | 13.9337 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3185 | 0.2155 | 0.5388 | 2.5937 | 10.7275 | 52.0090 |
| SIGNAL 2: | 0.2770 | 0.2549 | 0.6212 | 2.6437 | 10.7470 | 58.6234 |
| SIGNAL 3: | 0.3123 | 0.2733 | 0.5398 | 2.9106 | 8.9611 | 54.0690 |
| SIGNAL 4: | 0.3022 | 0.2595 | 0.6690 | 2.7903 | 9.6474 | 53.2208 |
| SIGNAL 5: | 0.2388 | 0.2737 | 0.5984 | 2.6087 | 10.4443 | 58.9315 |
| SIGNAL 6: | 0.2715 | 0.2588 | 0.5428 | 2.5480 | 9.9815 | 52.8461 |
| SIGNAL 7: | 0.2536 | 0.2180 | 0.5879 | 2.3883 | 10.0475 | 54.4567 |
| SIGNAL 8: | 0.2530 | 0.2886 | 0.5516 | 2.6977 | 10.0750 | 50.2555 |
| SIGNAL 9: | 0.2634 | 0.2249 | 0.4667 | 2.5582 | 7.2510 | 43.6765 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| SIGNAL 2: | 0.0145 | 0.0182 | 0.1264 | 0.1059 | 0.2600 | 2.5576 |
| SIGNAL 3: | 0.0140 | 0.0574 | 0.0679 | 0.1825 | 0.5453 | 1.9164 |
| SIGNAL 4: | 0.0087 | 0.0243 | 0.1422 | 0.0815 | 0.2531 | 0.5718 |
| SIGNAL 5: | 0.0202 | 0.0205 | 0.1187 | 0.0708 | 0.2221 | 1.8509 |
| SIGNAL 6: | 0.0173 | 0.0460 | 0.0522 | 0.1326 | 0.5117 | 1.8301 |
| SIGNAL 7: | 0.0080 | 0.0164 | 0.2044 | 0.1456 | 0.2368 | 0.6173 |
| SIGNAL 8: | 0.0202 | 0.0689 | 0.0991 | 0.1083 | 0.3890 | 1.7340 |
| SIGNAL 9: | 0.0165 | 0.0355 | 0.0718 | 0.1247 | 1.3389 | 2.9981 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3651 | 0.5208 | 1.1902 | 5.0321 | 23.6918 | 115.5311 |
| SIGNAL 2: | 0.3238 | 0.5603 | 1.3472 | 5.0435 | 23.8257 | 122.6571 |
| SIGNAL 3: | 0.3244 | 0.5725 | 1.2440 | 5.3595 | 22.0900 | 119.5437 |
| SIGNAL 4: | 0.3540 | 0.5550 | 1.3640 | 5.2345 | 22.9700 | 116.3537 |
| SIGNAL 5: | 0.2868 | 0.5780 | 1.3037 | 5.0934 | 23.5456 | 124.5405 |
| SIGNAL 6: | 0.3084 | 0.5596 | 1.2133 | 4.8809 | 23.0578 | 117.4324 |
| SIGNAL 7: | 0.2942 | 0.5274 | 1.3532 | 4.6602 | 23.2007 | 118.0311 |
| SIGNAL 8: | 0.2827 | 0.5685 | 1.2326 | 5.0956 | 23.2144 | 113.1257 |
| SIGNAL 9: | 0.2940 | 0.5262 | 1.0909 | 4.9186 | 19.2677 | 104.3332 |

D. SIGNALS FROM TRANSMITTER 4 ARE THE INPUTS

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3094 | 0.5771 | 1.2600 | 4.7861 | 23.4442 | 107.8438 |
| SIGNAL 2: | 0.3736 | 0.9836 | 3.0273 | 9.2057 | 38.3416 | 164.0387 |
| SIGNAL 3: | 0.3331 | 0.6218 | 1.3112 | 5.3833 | 24.3415 | 113.3645 |
| SIGNAL 4: | 0.3133 | 0.5338 | 1.3146 | 5.2073 | 24.1267 | 112.2255 |
| SIGNAL 5: | 0.4374 | 0.5831 | 1.1027 | 5.0784 | 23.9667 | 107.1038 |
| SIGNAL 6: | 0.3432 | 0.5722 | 1.2066 | 5.4055 | 23.0626 | 110.6970 |
| SIGNAL 7: | 0.2560 | 0.4075 | 0.8567 | 3.8320 | 18.1956 | 83.7372 |
| SIGNAL 8: | 0.2761 | 0.4673 | 1.3469 | 4.5214 | 28.6522 | 133.7564 |
| SIGNAL 9: | 0.2699 | 0.6246 | 1.7105 | 7.5262 | 30.5762 | 136.1710 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0688 | 0.3144 | 0.7227 | 2.4151 | 10.7828 | 46.0266 |
| SIGNAL 2: | 0.2104 | 0.6746 | 1.6970 | 4.8096 | 18.7294 | 72.9213 |
| SIGNAL 3: | 0.1649 | 0.2812 | 0.8120 | 2.6968 | 11.2046 | 50.4846 |
| SIGNAL 4: | 0.1370 | 0.2327 | 0.8829 | 2.6756 | 11.2253 | 57.7131 |
| SIGNAL 5: | 0.1847 | 0.3027 | 0.5789 | 2.4825 | 10.9737 | 45.7909 |
| SIGNAL 6: | 0.1039 | 0.2557 | 0.6717 | 2.8612 | 10.1952 | 53.3557 |
| SIGNAL 7: | 0.1637 | 0.1727 | 0.5083 | 2.1826 | 6.2022 | 38.8847 |
| SIGNAL 8: | 0.1866 | 0.2580 | 0.8640 | 2.9625 | 12.0382 | 54.5502 |
| SIGNAL 9: | 0.1472 | 0.3361 | 0.9322 | 4.4756 | 14.4394 | 55.8276 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3651 | 0.5208 | 1.1902 | 5.0321 | 23.6918 | 115.5311 |
| SIGNAL 2: | 0.4188 | 0.8757 | 2.3272 | 8.7571 | 35.0247 | 158.7921 |
| SIGNAL 3: | 0.2974 | 0.5532 | 1.0881 | 5.5421 | 24.5287 | 120.8973 |
| SIGNAL 4: | 0.3393 | 0.4954 | 1.1246 | 5.4472 | 24.5602 | 122.9673 |
| SIGNAL 5: | 0.4295 | 0.5159 | 1.0414 | 5.1931 | 24.3348 | 114.7444 |
| SIGNAL 6: | 0.3967 | 0.5028 | 1.0955 | 5.6244 | 23.1653 | 119.4670 |
| SIGNAL 7: | 0.2582 | 0.3225 | 0.7155 | 4.1114 | 18.9958 | 94.1898 |
| SIGNAL 8: | 0.2952 | 0.3749 | 1.1948 | 5.2218 | 25.6827 | 132.6012 |
| SIGNAL 9: | 0.2697 | 0.5467 | 1.3014 | 6.8545 | 27.5281 | 133.7723 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| SIGNAL 2: | 0.1732 | 0.5275 | 1.2713 | 3.4912 | 9.3186 | 30.3546 |
| SIGNAL 3: | 0.0544 | 0.0466 | 0.3896 | 0.4135 | 2.4614 | 6.5196 |
| SIGNAL 4: | 0.0610 | 0.0694 | 0.4249 | 0.3959 | 2.6097 | 14.8425 |
| SIGNAL 5: | 0.0986 | 0.0655 | 0.1514 | 0.4633 | 2.2296 | 6.8575 |
| SIGNAL 6: | 0.0528 | 0.0506 | 0.0990 | 0.5903 | 3.7932 | 11.8557 |
| SIGNAL 7: | 0.0814 | 0.1619 | 0.4059 | 1.7605 | 4.4264 | 22.1434 |
| SIGNAL 8: | 0.0935 | 0.3151 | 0.6468 | 2.8861 | 13.5405 | 47.6043 |
| SIGNAL 9: | 0.0776 | 0.3200 | 0.9673 | 3.8654 | 13.8608 | 49.1905 |

**E. SIGNALS FROM TRANSMITTER 1 ARE THE INPUTS:
10 DB SNR REDUCTION**

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0375 | 0.1043 | 0.2759 | 0.7683 | 1.3094 | 3.3502 |
| SIGNAL 2: | 0.0468 | 0.1085 | 0.3193 | 0.7442 | 2.0640 | 3.6190 |
| SIGNAL 3: | 0.0431 | 0.0800 | 0.2972 | 0.6296 | 1.3008 | 3.5047 |
| SIGNAL 4: | 0.0504 | 0.1064 | 0.3262 | 0.7624 | 1.7088 | 3.4416 |
| SIGNAL 5: | 0.0421 | 0.0706 | 0.3415 | 0.6067 | 1.5736 | 5.5176 |
| SIGNAL 6: | 0.0508 | 0.0743 | 0.3502 | 0.8021 | 1.5439 | 5.0661 |
| SIGNAL 7: | 0.0291 | 0.1163 | 0.3047 | 0.6125 | 1.2903 | 3.8559 |
| SIGNAL 8: | 0.0377 | 0.1302 | 0.2676 | 0.5524 | 1.3807 | 3.7544 |
| SIGNAL 9: | 0.0337 | 0.1012 | 0.2747 | 0.5609 | 1.5242 | 5.8662 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3553 | 0.2728 | 0.6642 | 2.8099 | 10.4138 | 47.5912 |
| SIGNAL 2: | 0.2227 | 0.2168 | 0.6260 | 2.7765 | 11.2037 | 48.0249 |
| SIGNAL 3: | 0.3561 | 0.2650 | 0.5283 | 2.4908 | 9.4202 | 43.7669 |
| SIGNAL 4: | 0.2451 | 0.2573 | 0.7072 | 2.5680 | 10.7271 | 46.0365 |
| SIGNAL 5: | 0.2911 | 0.2100 | 0.5792 | 2.2985 | 9.2627 | 42.8591 |
| SIGNAL 6: | 0.2959 | 0.2452 | 0.6188 | 2.8187 | 10.3953 | 49.7207 |
| SIGNAL 7: | 0.2792 | 0.3019 | 0.5697 | 2.5446 | 8.9549 | 42.6443 |
| SIGNAL 8: | 0.3114 | 0.2584 | 0.5708 | 2.5522 | 9.9918 | 43.5053 |
| SIGNAL 9: | 0.3268 | 0.2492 | 0.5431 | 2.2212 | 9.0768 | 42.7932 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0359 | 0.1303 | 0.3973 | 1.0658 | 3.8188 | 15.6111 |
| SIGNAL 2: | 0.0389 | 0.0850 | 0.3844 | 1.0643 | 4.3243 | 16.0790 |
| SIGNAL 3: | 0.0430 | 0.0984 | 0.3287 | 1.0871 | 4.0226 | 16.3021 |
| SIGNAL 4: | 0.0373 | 0.1119 | 0.3942 | 1.0760 | 3.9323 | 16.3871 |
| SIGNAL 5: | 0.0346 | 0.0814 | 0.3848 | 1.1143 | 4.0585 | 15.6347 |
| SIGNAL 6: | 0.0390 | 0.0843 | 0.4338 | 0.9025 | 3.6642 | 14.2155 |
| SIGNAL 7: | 0.0231 | 0.1120 | 0.3312 | 1.1282 | 4.3698 | 17.4460 |
| SIGNAL 8: | 0.0353 | 0.1325 | 0.3284 | 1.0011 | 4.0881 | 16.2695 |
| SIGNAL 9: | 0.0315 | 0.1139 | 0.3366 | 1.0887 | 3.4706 | 13.8132 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3328 | 0.5612 | 1.3761 | 4.9135 | 22.4741 | 107.4480 |
| SIGNAL 2: | 0.2562 | 0.4936 | 1.2892 | 4.8175 | 22.7738 | 102.1320 |
| SIGNAL 3: | 0.3450 | 0.5801 | 1.1919 | 4.5966 | 20.6796 | 100.6919 |
| SIGNAL 4: | 0.3263 | 0.5714 | 1.3486 | 4.7544 | 23.1659 | 106.4092 |
| SIGNAL 5: | 0.3633 | 0.5636 | 1.2499 | 4.3789 | 20.7572 | 97.1743 |
| SIGNAL 6: | 0.3042 | 0.5860 | 1.3593 | 5.1565 | 22.9280 | 115.2724 |
| SIGNAL 7: | 0.3338 | 0.4590 | 1.2041 | 4.6390 | 20.0266 | 96.3665 |
| SIGNAL 8: | 0.3610 | 0.5003 | 1.2423 | 4.3976 | 21.1563 | 98.0142 |
| SIGNAL 9: | 0.3608 | 0.5617 | 1.2187 | 4.3478 | 21.0854 | 100.2342 |

**F. SIGNALS FROM TRANSMITTER 2 ARE INPUTS:
10 DB SNR REDUCTION**

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.2998 | 0.2975 | 0.6936 | 2.5272 | 9.2397 | 41.5986 |
| SIGNAL 2: | 0.1149 | 0.2315 | 0.4739 | 2.1895 | 9.2329 | 42.4683 |
| SIGNAL 3: | 0.2483 | 0.3285 | 0.6071 | 2.1489 | 9.2771 | 44.6032 |
| SIGNAL 4: | 0.1683 | 0.2955 | 0.6568 | 2.4697 | 9.9201 | 46.1952 |
| SIGNAL 5: | 0.2497 | 0.3499 | 0.6493 | 2.6333 | 10.4630 | 46.5821 |
| SIGNAL 6: | 0.1699 | 0.3207 | 0.6972 | 2.3188 | 9.9021 | 43.4716 |
| SIGNAL 7: | 0.2107 | 0.4434 | 0.8268 | 2.5254 | 10.3927 | 45.1681 |
| SIGNAL 8: | 0.3312 | 0.3403 | 0.8018 | 2.6353 | 10.2630 | 47.7992 |
| SIGNAL 9: | 0.3012 | 0.4205 | 0.7371 | 2.3549 | 10.5009 | 47.2822 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0353 | 0.1029 | 0.3632 | 1.0898 | 3.2442 | 9.5561 |
| SIGNAL 2: | 0.1267 | 0.1354 | 0.2513 | 0.9447 | 2.8265 | 8.8378 |
| SIGNAL 3: | 0.0796 | 0.1694 | 0.4690 | 0.6193 | 3.2228 | 9.6555 |
| SIGNAL 4: | 0.1472 | 0.1665 | 0.4090 | 0.8105 | 7.2229 | 14.2345 |
| SIGNAL 5: | 0.0873 | 0.1667 | 0.2662 | 0.9486 | 2.9736 | 13.1501 |
| SIGNAL 6: | 0.1078 | 0.1217 | 0.4735 | 1.5780 | 5.9286 | 21.2568 |
| SIGNAL 7: | 0.1093 | 0.1754 | 0.6404 | 1.3555 | 8.2270 | 18.1313 |
| SIGNAL 8: | 0.0543 | 0.1232 | 0.5491 | 1.1825 | 6.1173 | 8.1226 |
| SIGNAL 9: | 0.0705 | 0.1886 | 0.4846 | 2.8504 | 9.8706 | 19.6026 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3378 | 0.2623 | 0.6906 | 2.3864 | 10.3773 | 48.5725 |
| SIGNAL 2: | 0.1748 | 0.2254 | 0.4875 | 2.3375 | 9.7415 | 46.8498 |
| SIGNAL 3: | 0.3212 | 0.3164 | 0.7229 | 2.2456 | 9.7959 | 48.5703 |
| SIGNAL 4: | 0.2274 | 0.3074 | 0.7264 | 2.4591 | 10.1376 | 49.7111 |
| SIGNAL 5: | 0.3371 | 0.3617 | 0.6446 | 2.5659 | 11.1484 | 51.7127 |
| SIGNAL 6: | 0.2661 | 0.2662 | 0.7978 | 2.5988 | 10.7795 | 48.9604 |
| SIGNAL 7: | 0.2570 | 0.3376 | 0.9129 | 2.2908 | 10.6121 | 47.4262 |
| SIGNAL 8: | 0.3939 | 0.3071 | 0.9439 | 2.4221 | 10.1326 | 54.1949 |
| SIGNAL 9: | 0.3870 | 0.3634 | 0.8302 | 2.3635 | 10.7260 | 49.2730 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.1007 | 0.2607 | 0.8255 | 2.5868 | 9.9048 | 44.4863 |
| SIGNAL 2: | 0.2222 | 0.2770 | 0.6216 | 2.0512 | 10.6348 | 48.9550 |
| SIGNAL 3: | 0.1201 | 0.3323 | 0.8504 | 2.2053 | 10.3880 | 45.7483 |
| SIGNAL 4: | 0.2068 | 0.3380 | 0.8050 | 2.4836 | 11.3920 | 47.6053 |
| SIGNAL 5: | 0.1445 | 0.2756 | 0.6615 | 2.7278 | 10.2407 | 51.0403 |
| SIGNAL 6: | 0.1370 | 0.3112 | 0.9905 | 2.0032 | 11.8533 | 53.8810 |
| SIGNAL 7: | 0.1369 | 0.3029 | 0.9008 | 2.7150 | 11.7296 | 53.8566 |
| SIGNAL 8: | 0.1439 | 0.2392 | 0.8706 | 3.2170 | 12.0501 | 46.7004 |
| SIGNAL 9: | 0.1384 | 0.3070 | 0.9407 | 3.0078 | 12.4255 | 52.4257 |

**G. SIGNALS FROM TRANSMITTER 3 ARE INPUTS:
10 DB SNR REDUCTION**

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0478 | 0.1274 | 0.5144 | 1.1785 | 4.0000 | 18.1846 |
| SIGNAL 2: | 0.0365 | 0.1234 | 0.4115 | 1.2813 | 4.1553 | 18.5676 |
| SIGNAL 3: | 0.0684 | 0.1219 | 0.3577 | 1.1853 | 3.8279 | 16.0824 |
| SIGNAL 4: | 0.0523 | 0.1170 | 0.4138 | 1.1140 | 4.3548 | 18.4488 |
| SIGNAL 5: | 0.0348 | 0.1336 | 0.5074 | 1.2930 | 3.9845 | 17.9401 |
| SIGNAL 6: | 0.0550 | 0.1275 | 0.4415 | 1.1653 | 3.9053 | 18.0756 |
| SIGNAL 7: | 0.0478 | 0.1284 | 0.4557 | 1.1081 | 3.6589 | 17.1620 |
| SIGNAL 8: | 0.0531 | 0.0784 | 0.3782 | 1.1528 | 4.0583 | 17.6531 |
| SIGNAL 9: | 0.0320 | 0.1208 | 0.3447 | 1.0956 | 3.6144 | 16.3529 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.2735 | 0.2305 | 0.8375 | 3.2786 | 12.6610 | 59.1120 |
| SIGNAL 2: | 0.2489 | 0.2555 | 0.7085 | 3.4069 | 13.2948 | 64.9324 |
| SIGNAL 3: | 0.2403 | 0.2307 | 0.6814 | 3.2335 | 13.1578 | 56.7576 |
| SIGNAL 4: | 0.2543 | 0.2354 | 0.6180 | 3.1848 | 12.5049 | 61.7671 |
| SIGNAL 5: | 0.2546 | 0.1906 | 0.8137 | 3.4720 | 12.2753 | 62.2135 |
| SIGNAL 6: | 0.2131 | 0.1717 | 0.8292 | 3.7610 | 11.4564 | 59.3982 |
| SIGNAL 7: | 0.2271 | 0.2378 | 0.7183 | 2.9783 | 12.5854 | 62.4342 |
| SIGNAL 8: | 0.2246 | 0.2336 | 0.7436 | 3.1300 | 12.7692 | 62.2046 |
| SIGNAL 9: | 0.2461 | 0.2098 | 0.6158 | 2.7632 | 10.8813 | 49.2295 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0203 | 0.0815 | 0.6030 | 0.9418 | 2.5238 | 6.4598 |
| SIGNAL 2: | 0.0201 | 0.0950 | 0.4735 | 1.0146 | 3.0909 | 9.5653 |
| SIGNAL 3: | 0.0465 | 0.0833 | 0.3634 | 0.9566 | 2.7794 | 6.4284 |
| SIGNAL 4: | 0.0362 | 0.0954 | 0.4588 | 0.7535 | 2.6418 | 7.8841 |
| SIGNAL 5: | 0.0253 | 0.0951 | 0.5927 | 1.0052 | 2.3614 | 7.6033 |
| SIGNAL 6: | 0.0485 | 0.0933 | 0.5202 | 1.0480 | 1.7231 | 5.9440 |
| SIGNAL 7: | 0.0328 | 0.0992 | 0.5659 | 0.7845 | 2.2809 | 8.0012 |
| SIGNAL 8: | 0.0354 | 0.0650 | 0.4643 | 0.8565 | 2.4123 | 8.5680 |
| SIGNAL 9: | 0.0193 | 0.0851 | 0.3710 | 0.7626 | 2.3411 | 5.4612 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3287 | 0.5182 | 1.5587 | 5.8201 | 25.7509 | 122.5166 |
| SIGNAL 2: | 0.1893 | 0.5261 | 1.4451 | 5.8022 | 26.7385 | 129.8573 |
| SIGNAL 3: | 0.2463 | 0.5399 | 1.3549 | 5.8309 | 25.7889 | 119.4983 |
| SIGNAL 4: | 0.3308 | 0.5394 | 1.3779 | 5.5470 | 25.6568 | 127.0563 |
| SIGNAL 5: | 0.2908 | 0.4859 | 1.6405 | 6.0450 | 25.2253 | 127.3273 |
| SIGNAL 6: | 0.2010 | 0.4756 | 1.5749 | 6.3159 | 24.4335 | 122.7423 |
| SIGNAL 7: | 0.2183 | 0.5214 | 1.4519 | 5.5075 | 25.3703 | 126.9125 |
| SIGNAL 8: | 0.1728 | 0.4902 | 1.4167 | 5.4070 | 25.5855 | 126.2491 |
| SIGNAL 9: | 0.2523 | 0.5030 | 1.2223 | 5.0020 | 23.1458 | 109.1846 |

**H. SIGNALS FROM TRANSMITTER 4 ARE INPUTS:
10 DB SNR REDUCTION**

TEMPLATE 1

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3270 | 0.5990 | 1.6784 | 5.8691 | 25.0385 | 118.3022 |
| SIGNAL 2: | 0.3248 | 0.5290 | 1.7946 | 5.8942 | 23.5419 | 117.8467 |
| SIGNAL 3: | 0.3220 | 0.6044 | 1.5821 | 6.1968 | 25.1165 | 118.5933 |
| SIGNAL 4: | 0.4828 | 0.4838 | 1.1432 | 5.2652 | 22.8460 | 112.4434 |
| SIGNAL 5: | 0.3500 | 0.5793 | 1.2490 | 5.8876 | 23.7742 | 113.3160 |
| SIGNAL 6: | 0.2645 | 0.4183 | 1.0173 | 4.2269 | 18.5720 | 84.9252 |

TEMPLATE 2

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0850 | 0.3153 | 1.0099 | 3.3984 | 13.1845 | 62.8281 |
| SIGNAL 2: | 0.1570 | 0.2157 | 0.8610 | 3.0255 | 13.4155 | 53.4214 |
| SIGNAL 3: | 0.1514 | 0.2960 | 0.8447 | 3.6034 | 12.9318 | 50.1148 |
| SIGNAL 4: | 0.2097 | 0.2221 | 0.7012 | 2.9600 | 10.1786 | 54.1355 |
| SIGNAL 5: | 0.1014 | 0.3442 | 0.7740 | 2.8104 | 13.7426 | 49.5709 |
| SIGNAL 6: | 0.1545 | 0.2012 | 0.7999 | 2.0320 | 8.7308 | 34.0015 |

TEMPLATE 3

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.3780 | 0.5205 | 1.5290 | 5.9429 | 24.3696 | 121.4879 |
| SIGNAL 2: | 0.3014 | 0.4644 | 1.5550 | 5.8505 | 22.9977 | 121.4152 |
| SIGNAL 3: | 0.3627 | 0.5140 | 1.4117 | 6.1926 | 24.7024 | 123.3364 |
| SIGNAL 4: | 0.4693 | 0.4223 | 1.0334 | 5.4432 | 23.3718 | 117.9020 |
| SIGNAL 5: | 0.3955 | 0.5023 | 1.1421 | 5.9523 | 24.0663 | 119.0966 |
| SIGNAL 6: | 0.2624 | 0.3113 | 0.9258 | 4.2049 | 18.3186 | 87.9338 |

TEMPLATE 4

| | SCALE 6 | SCALE 7 | SCALE 8 | SCALE 9 | SCALE 10 | SCALE 11 |
|-----------|---------|---------|---------|---------|----------|----------|
| SIGNAL 1: | 0.0328 | 0.0486 | 0.4642 | 1.0811 | 4.1788 | 23.1301 |
| SIGNAL 2: | 0.0725 | 0.1947 | 0.2994 | 0.9492 | 2.7563 | 11.3148 |
| SIGNAL 3: | 0.0789 | 0.0741 | 0.3058 | 1.0105 | 4.5781 | 9.8803 |
| SIGNAL 4: | 0.1255 | 0.2016 | 0.2156 | 0.7971 | 1.8608 | 17.1363 |
| SIGNAL 5: | 0.0563 | 0.0770 | 0.2064 | 1.9411 | 4.4024 | 11.8552 |
| SIGNAL 6: | 0.0804 | 0.2145 | 0.7304 | 1.9989 | 7.3588 | 29.4086 |

APPENDIX C

PREPROCESSED SIGNALS AND TEMPLATE OUTPUTS

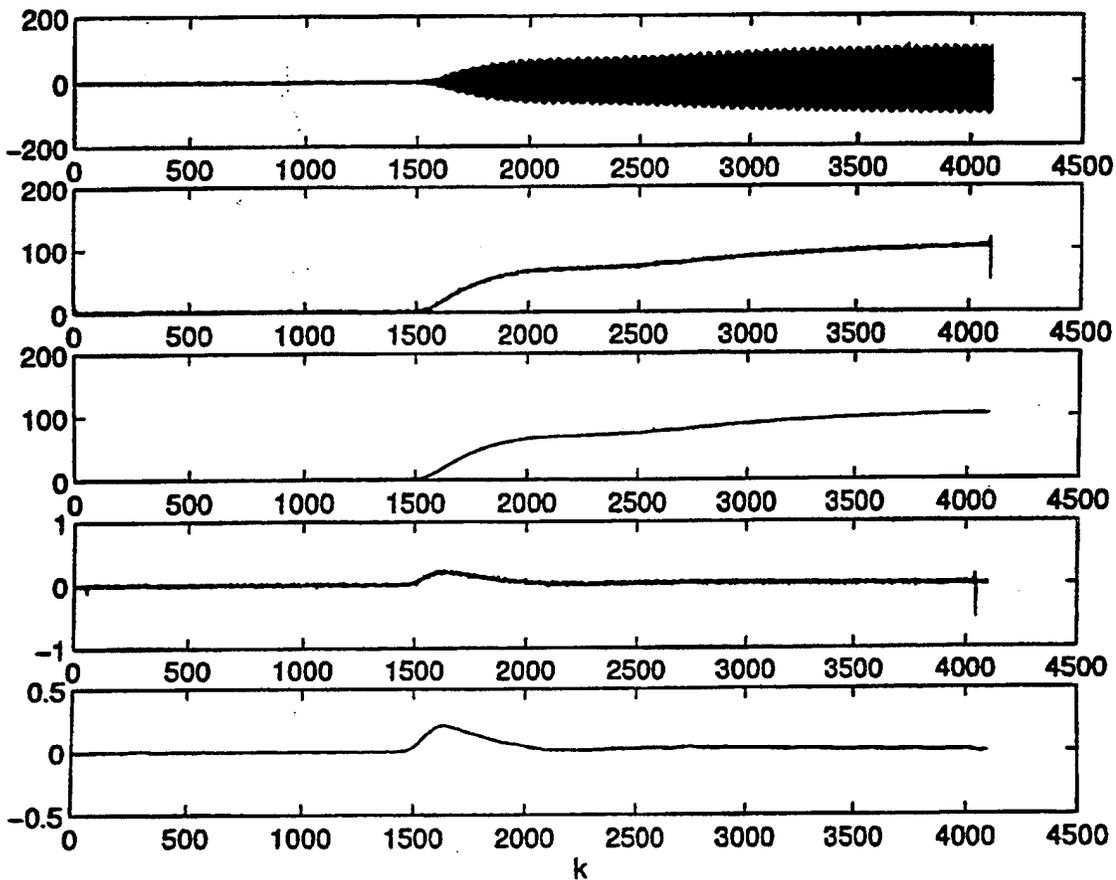


Figure C.1 Preprocessing scheme to obtain an appropriate waveform for Signal 1 of Transmitter 1. From top to bottom: Signal 1 of Transmitter 1 (d.c. removed); Upper envelope of Signal 1; Envelope after median filtering of size 100; Differential of median filtering output; Differential after median filtering of size 50.

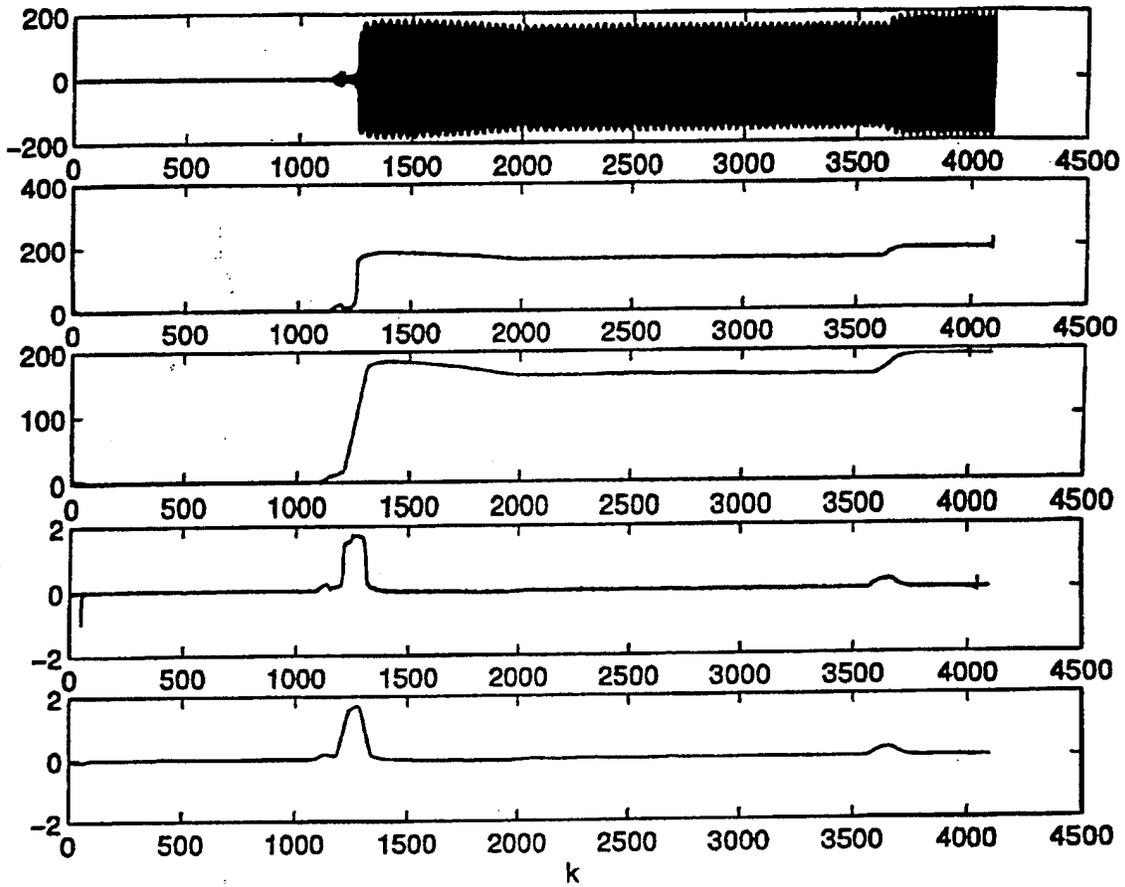


Figure C.2 Preprocessing scheme to obtain appropriate waveform for Signal 1 of Transmitter 2. From top to bottom: Signal 1 of Transmitter 2 (d.c. removed); Upper envelope of Signal 1; Envelope after median filtering of size 100; Differential of median filtering output; Differential after median filtering of size 50.

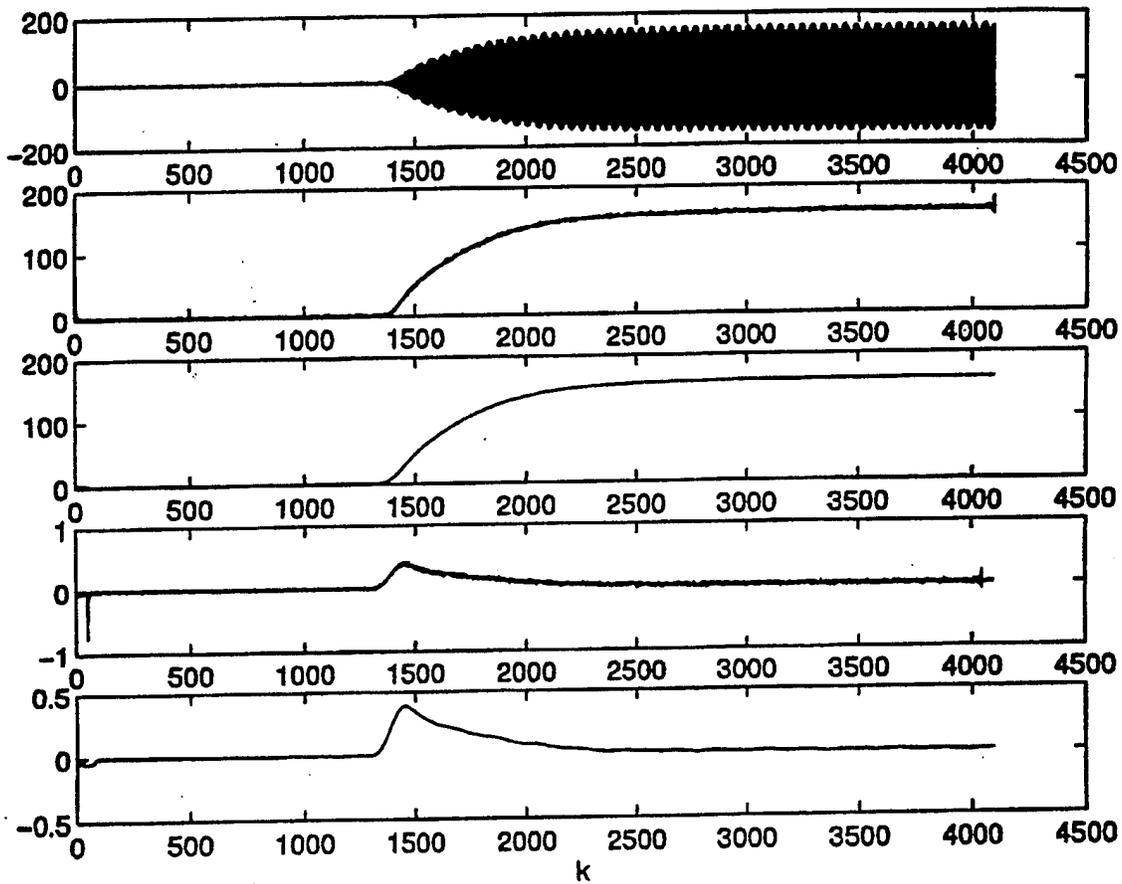


Figure C.3 Preprocessing scheme to obtain appropriate waveform for Signal 1 of Transmitter 3. From top to bottom: Signal 1 of Transmitter 3 (d.c. removed); Upper envelope of Signal 1; Envelope after median filtering of size 100; Differential of median filtering output; Differential after median filtering of size 50.

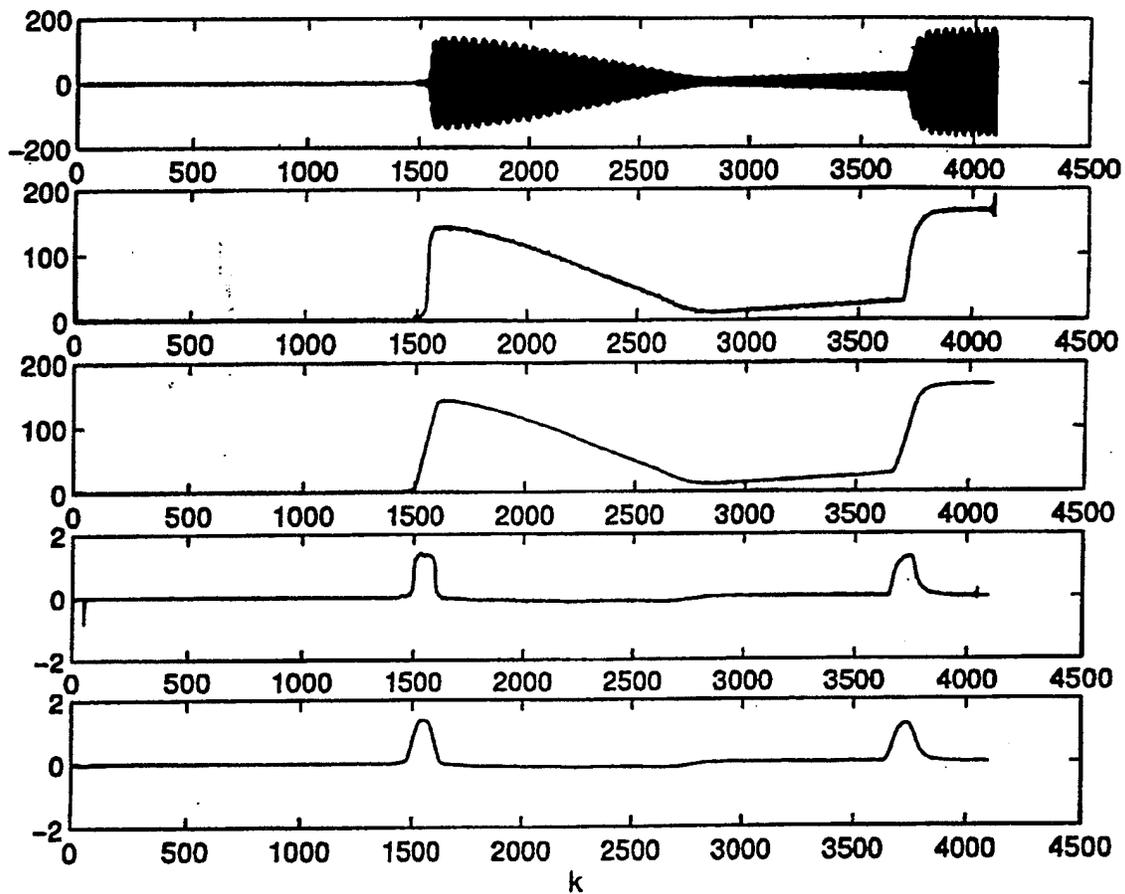


Figure C.4 Preprocessing scheme to obtain appropriate waveform for Signal 1 of Transmitter 4. From top to bottom: Signal 1 of Transmitter 4 (d.c. removed); Upper envelope of Signal 1; Envelope after median filtering of size 100; Differential of median filtering output; Differential after median filtering of size 50.

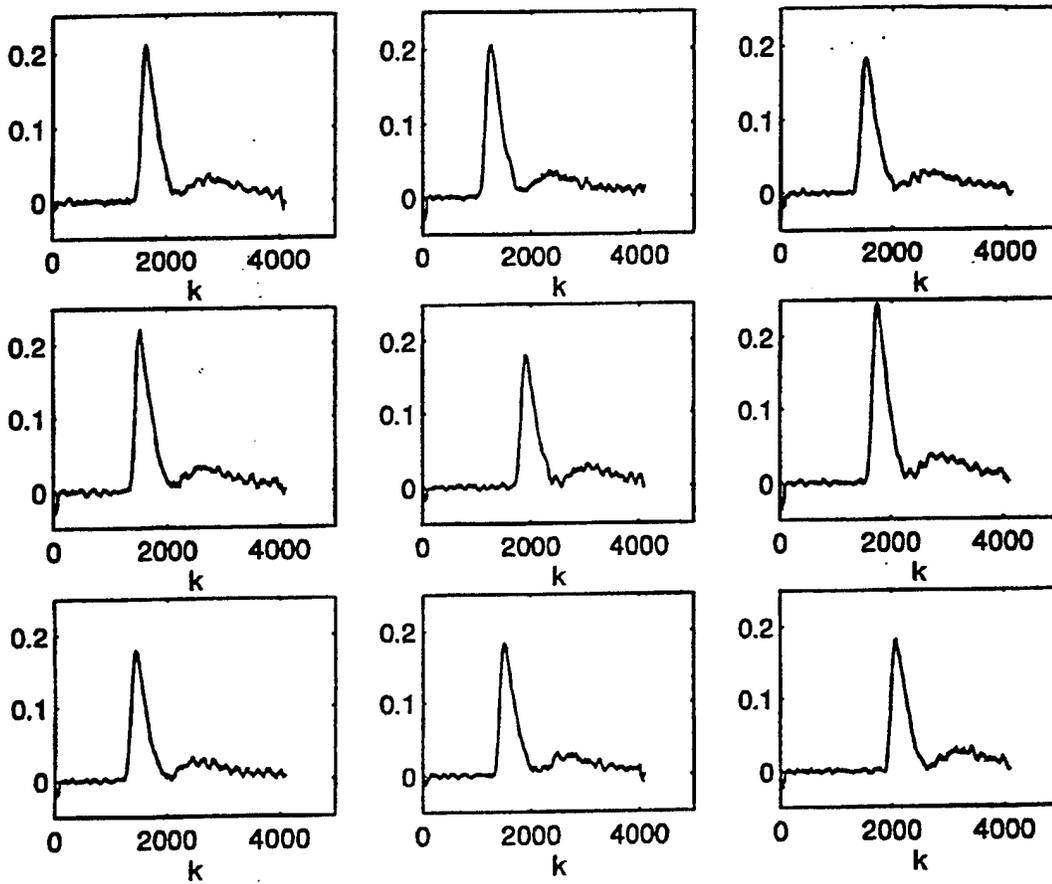


Figure C.5 Preprocessed signals from Transmitter 1.

Top row (left to right): Signal 1; Signal 2; Signal 3

Center row: Signal 4; Signal 5; Signal 6

Bottom row: Signal 7; Signal 8; Signal 9

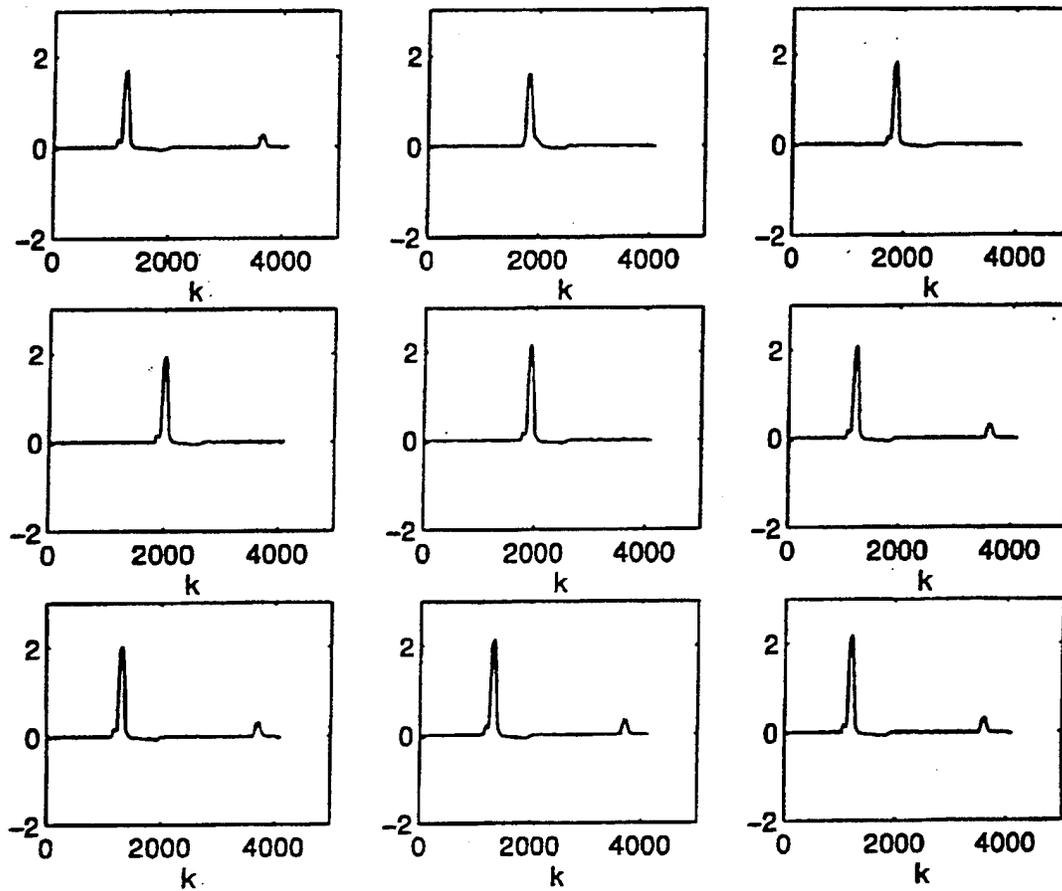


Figure C.6 Preprocessed signals from Transmitter 2.
 Top row (left to right): Signal 1; Signal 2; Signal 3
 Center row: Signal 4; Signal 5; Signal 6
 Bottom row: Signal 7; Signal 8; Signal 9

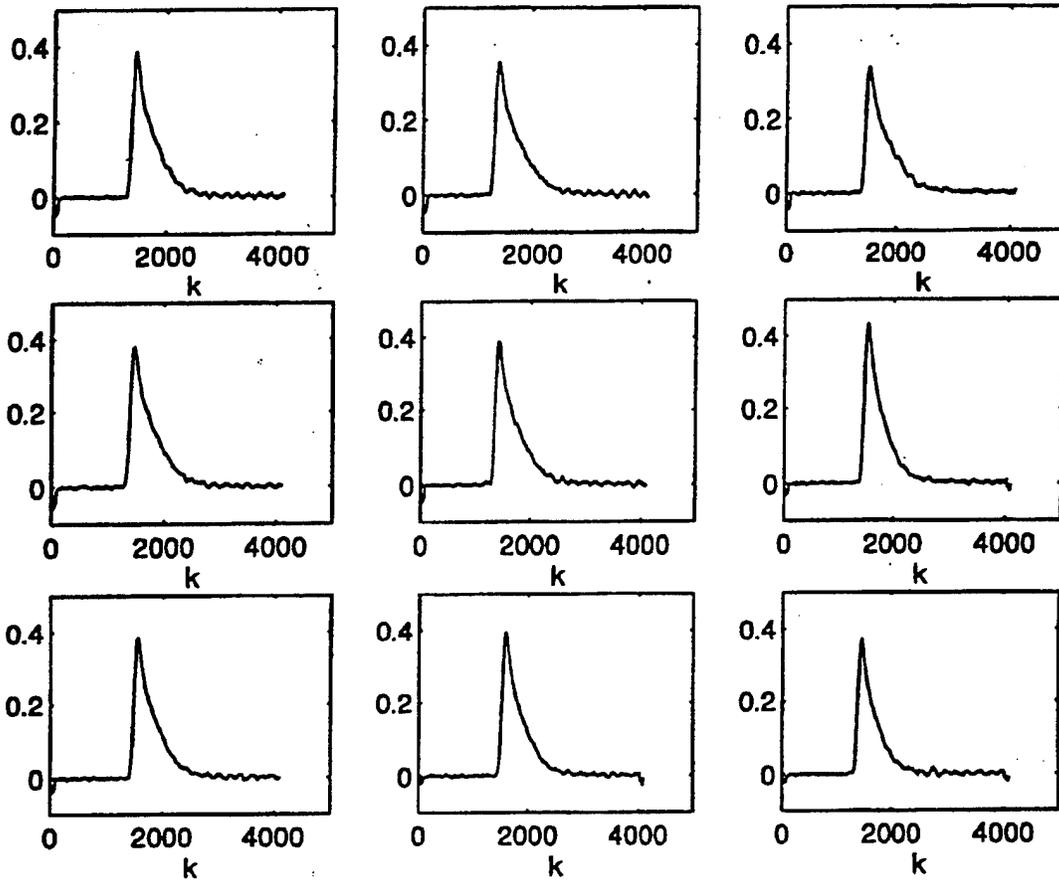


Figure C.7 Preprocessed signals from Transmitter 3.
 Top row (left to right): Signal 1; Signal 2; Signal 3
 Center row: Signal 4; Signal 5; Signal 6
 Bottom row: Signal 7; Signal 8; Signal 9

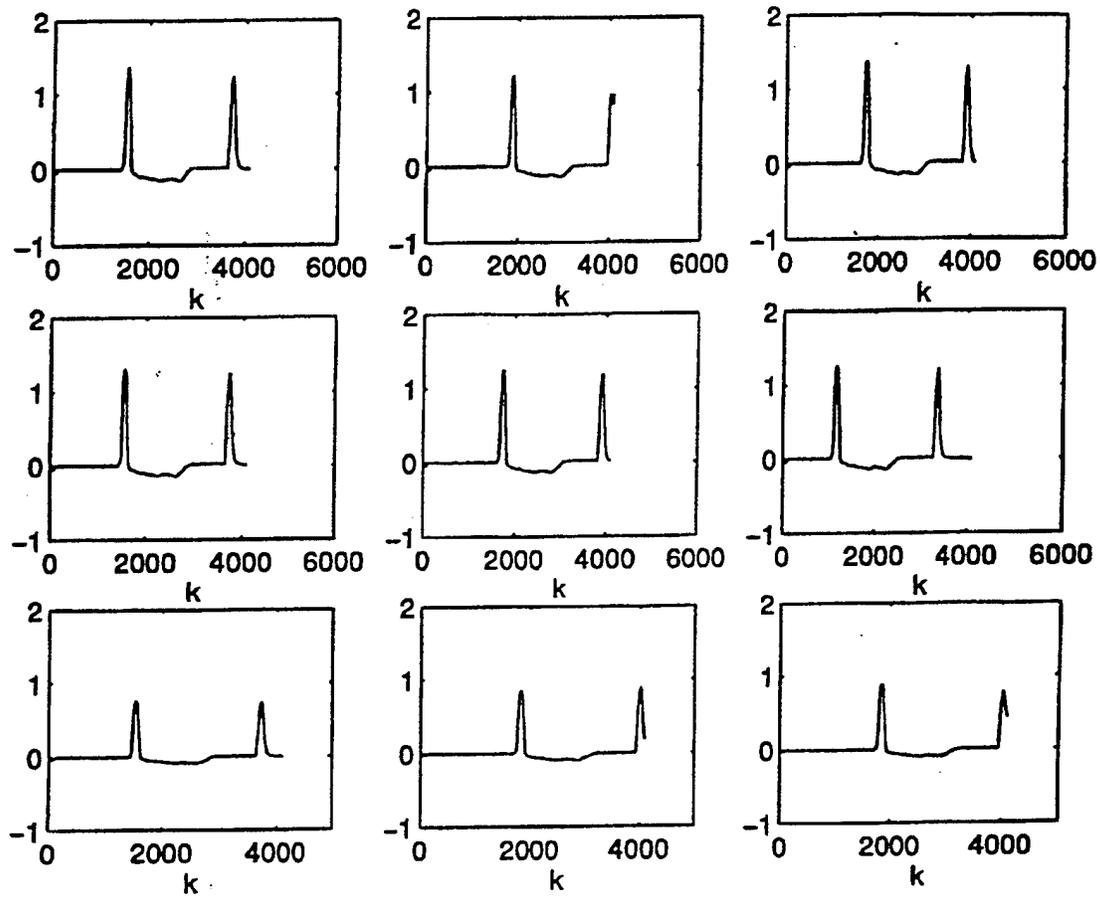


Figure C.8 Preprocessed signals from Transmitter 4.

Top row (left to right): Signal 1; Signal 2; Signal 3

Center row: Signal 4; Signal 5; Signal 6

Bottom row: Signal 7; Signal 8; Signal 9

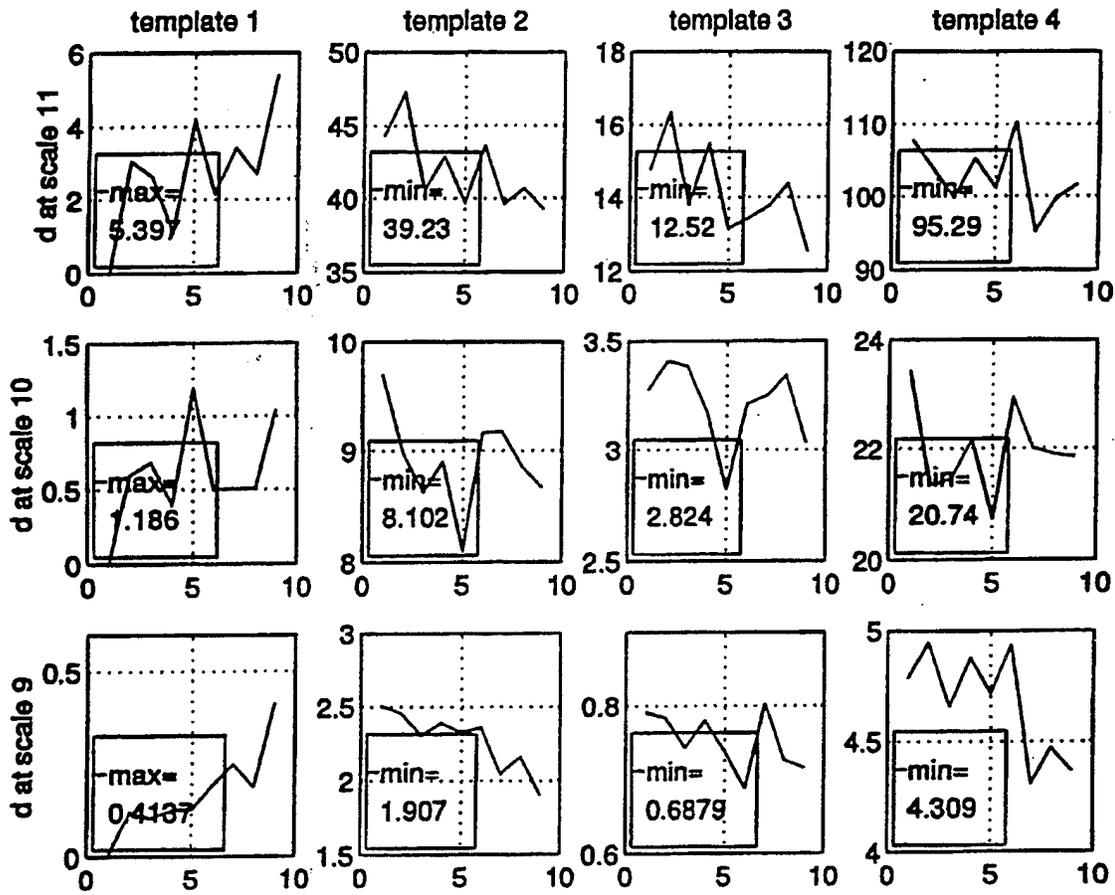


Figure C.9 Distance measures at the output of templates when signals from Transmitter 1 are the inputs. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

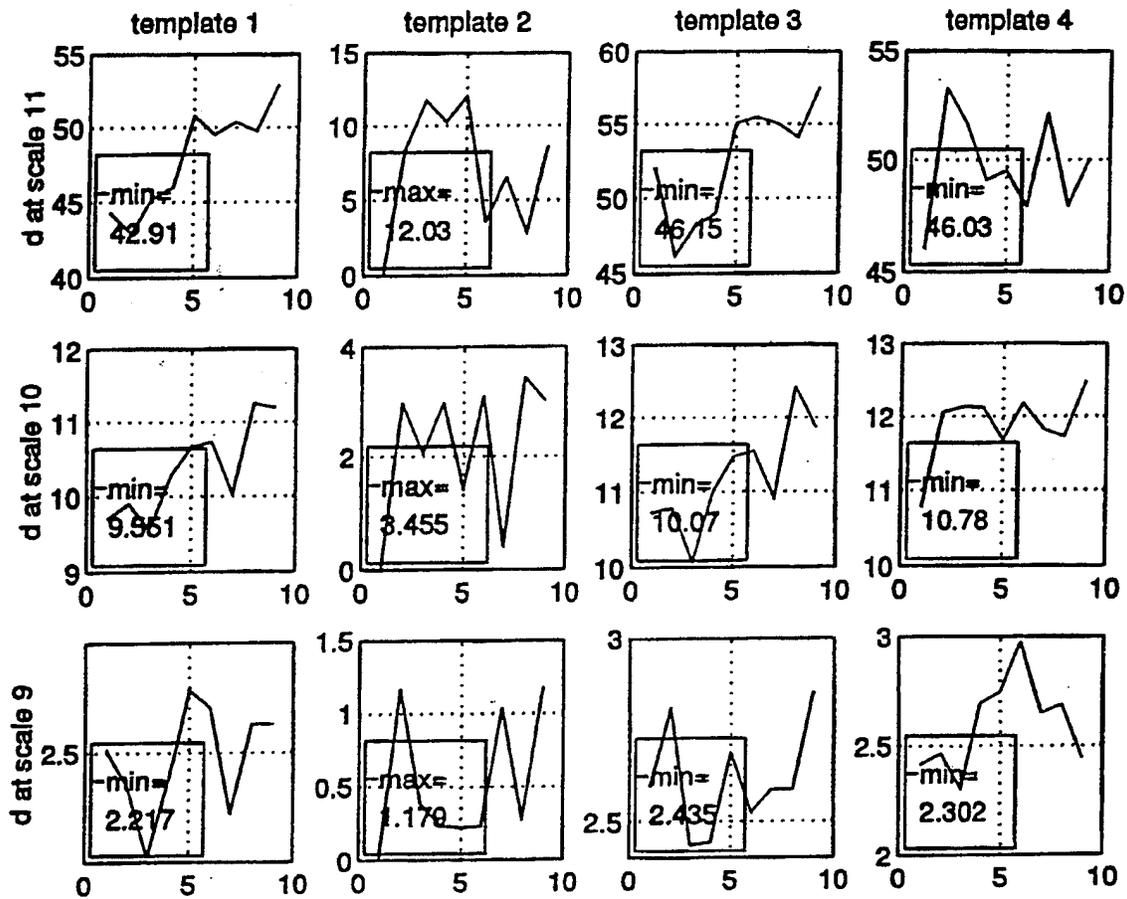


Figure C.10 Distance measures at the output of templates when signals from Transmitter 2 are the inputs. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

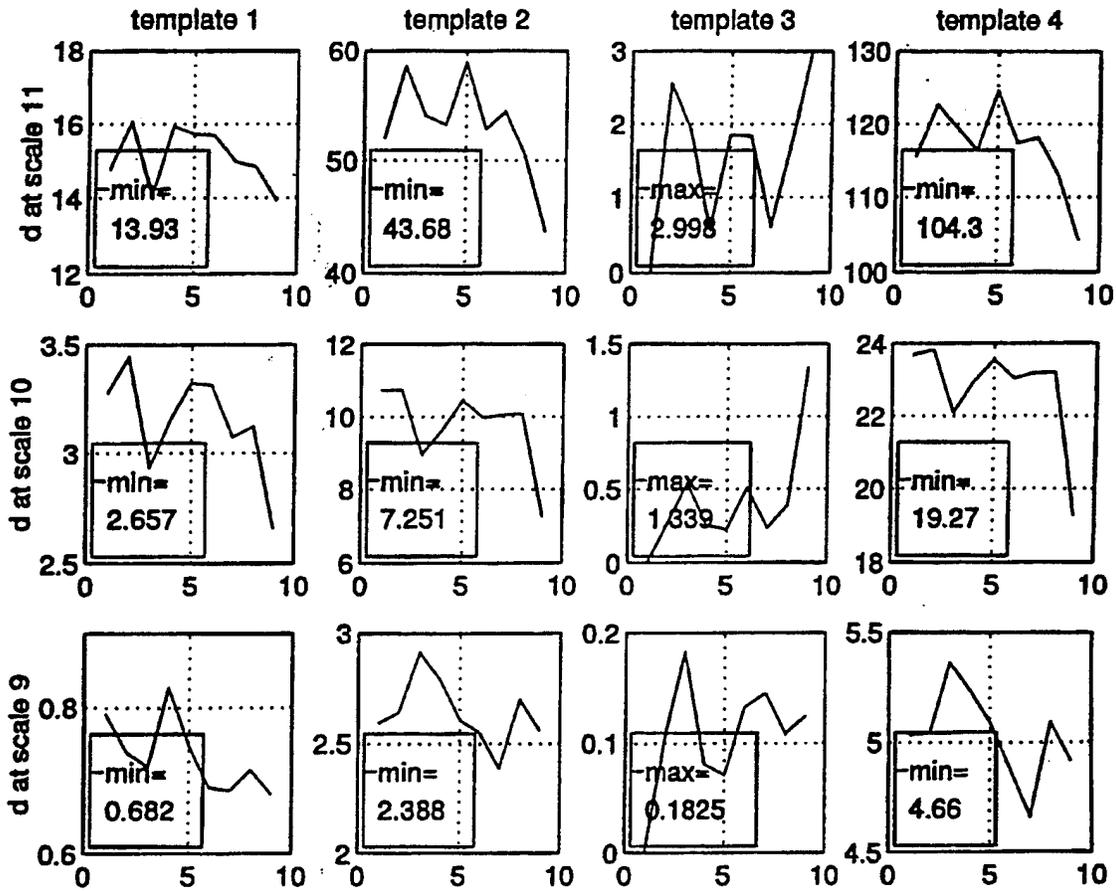


Figure C.11 Distance measures at the output of templates when signals from Transmitter 3 are the inputs. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

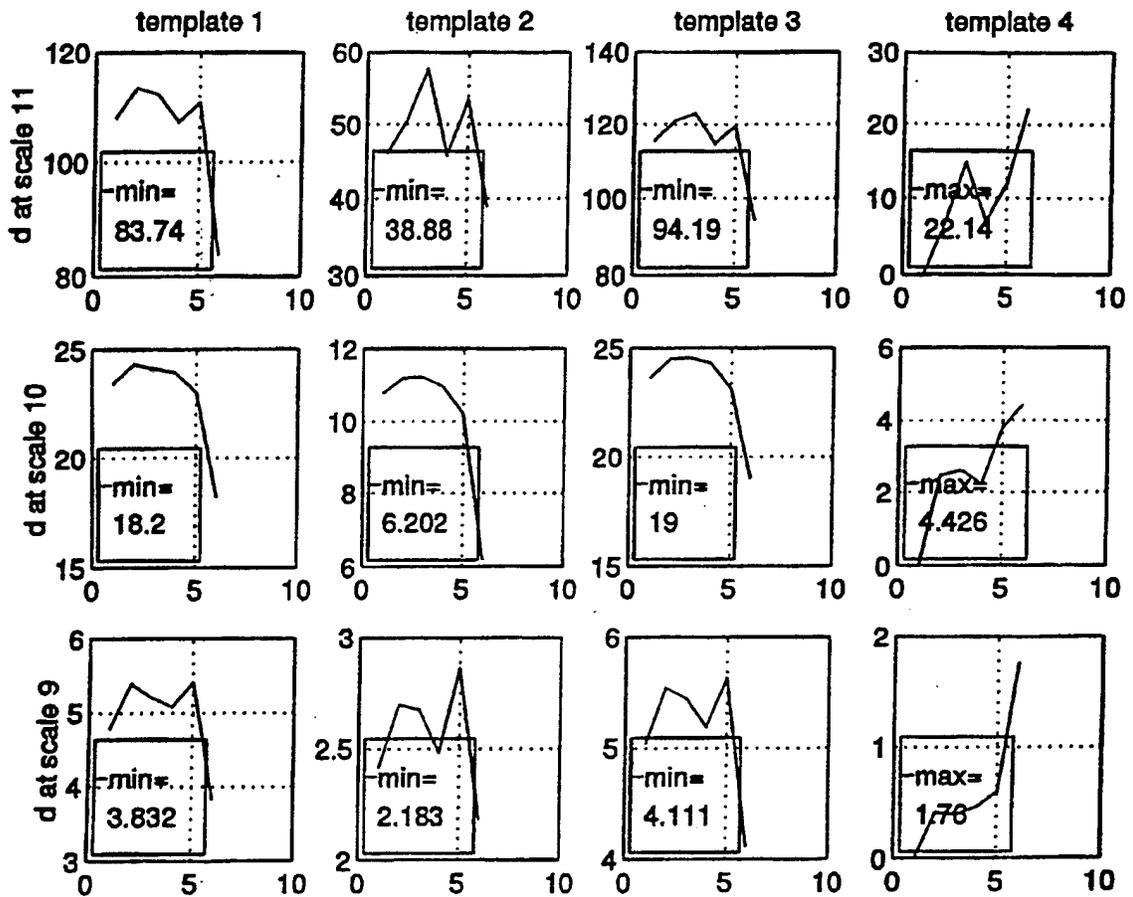


Figure C.12 Distance measures when signals from Transmitter 4 are the inputs. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

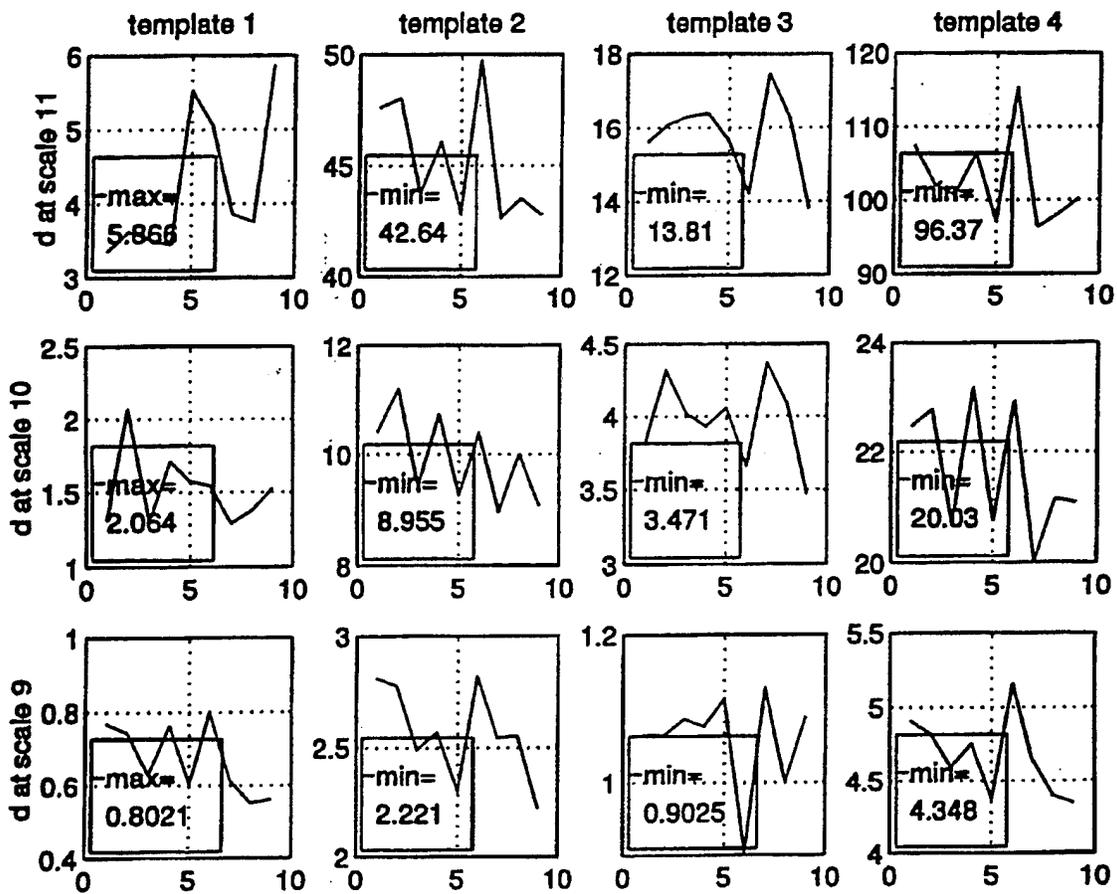


Figure C.13 Distance measures when signals from Transmitter 1 are inputs and their SNR values are 10 dB lower than the original data set. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

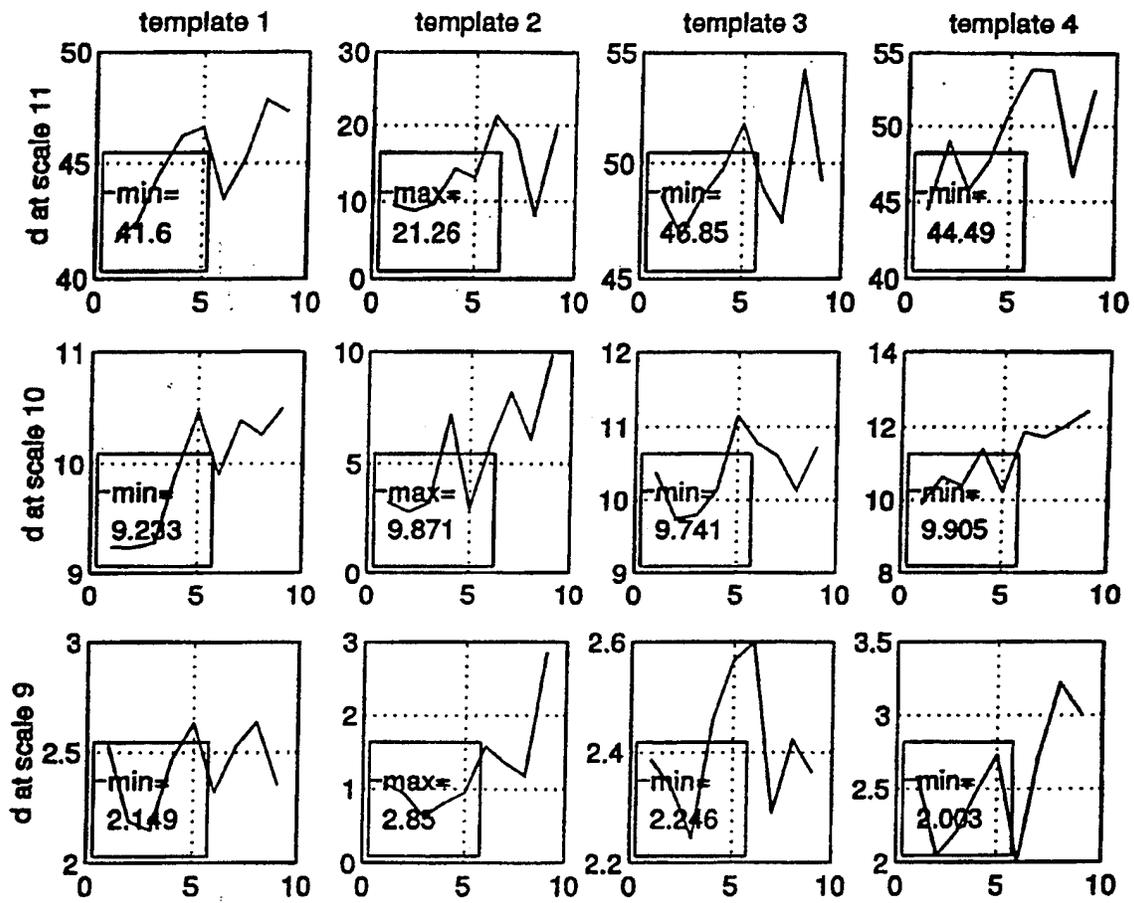


Figure C.14 Distance measures when signals from Transmitter 2 are inputs and their SNR values are 10 dB lower than the original data set. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

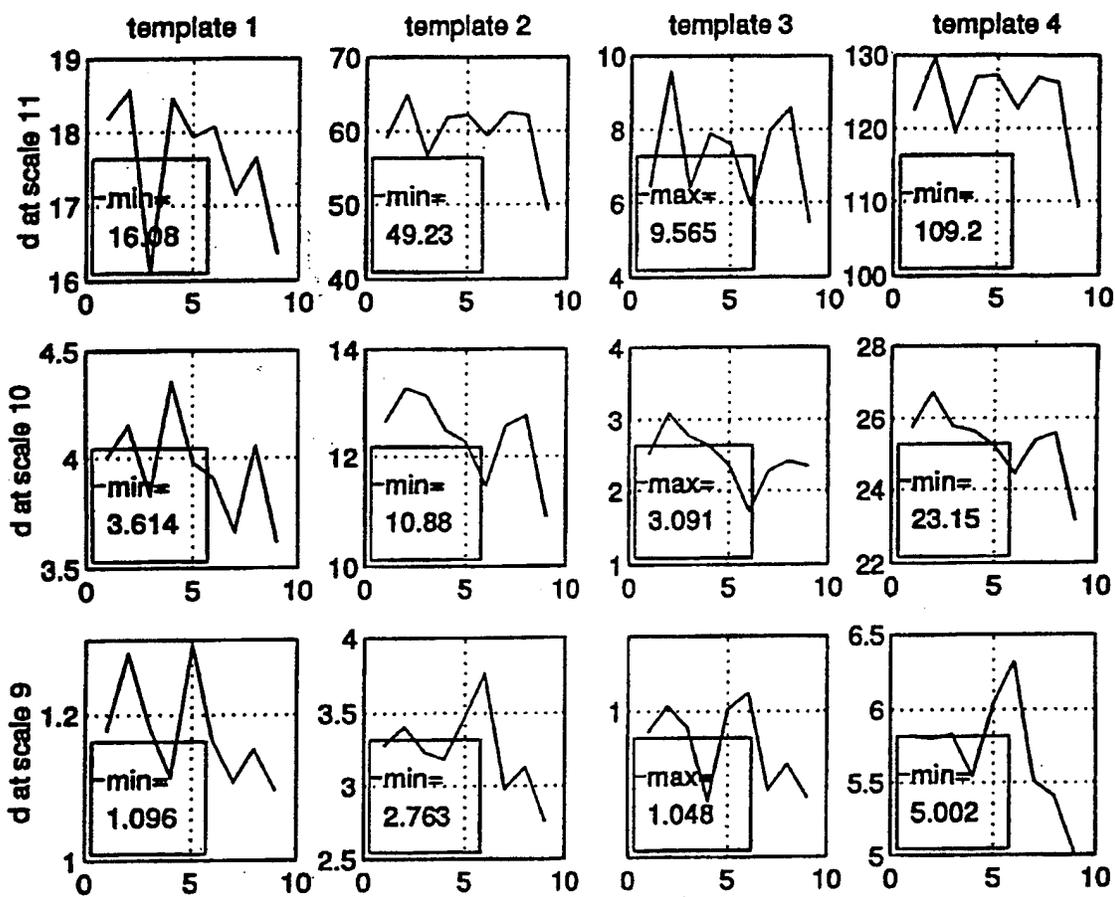


Figure C.15 Distance measures when signals from Transmitter 3 are inputs and their SNR values are 10 dB lower than the original data set. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

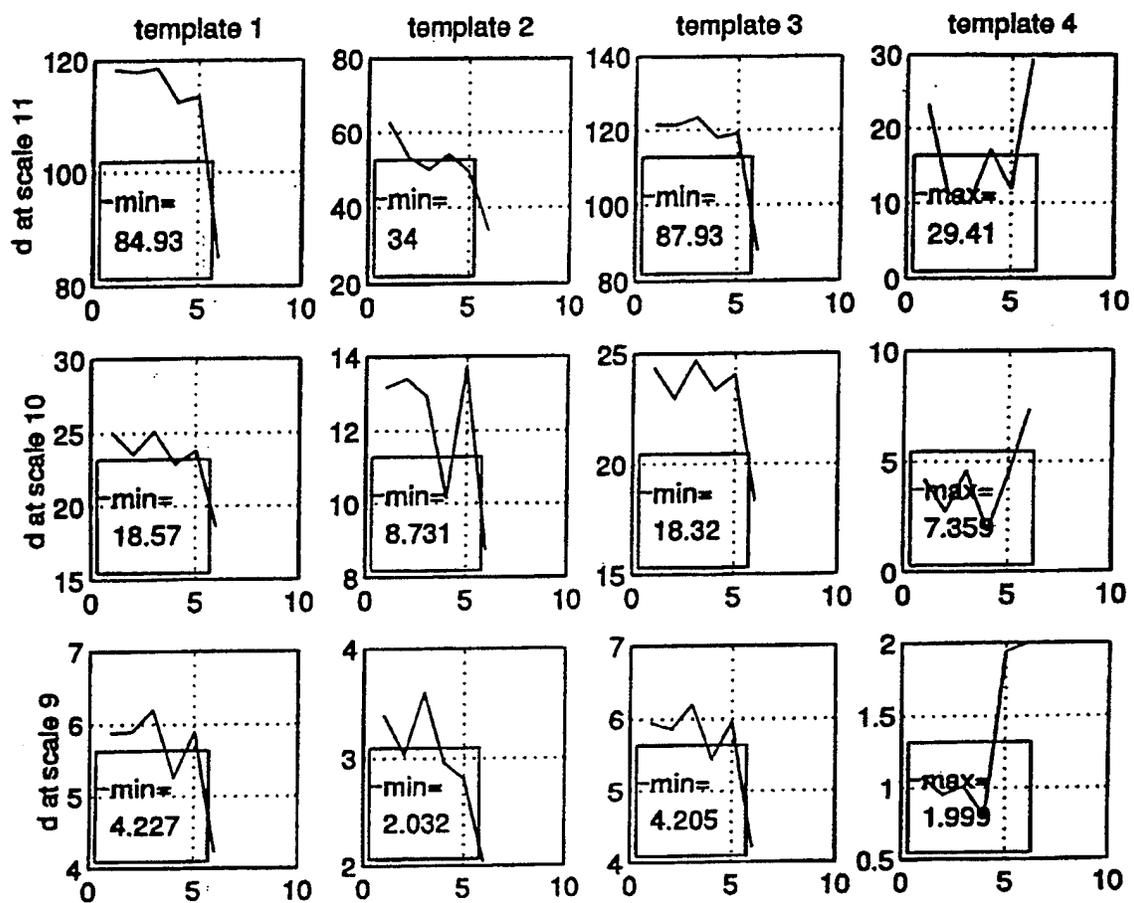


Figure C.16 Distance measures when signals from Transmitter 4 are inputs and their SNR values are 10 dB lower than the original data set. Horizontal axis shows the number of the signals. Each column represents a template. First row is for Scale 11; second row is for Scale 10; third row is for Scale 9.

APPENDIX D

MATLAB CODES

wtsignal.m

Purpose:

To transform the push-to-talk recordings into pulse shape forms which are suitable for wavelet analysis.

Synopsis:

wtsignal

Description:

The program preprocesses the data as mentioned in Chapter 3. The signal vector to be preprocessed is entered as the input.

Created by Yalcin Payal, November, 1995.

```
x=input('enter the signal to be preprocessed = ')
x=x-mean(x);
y=envelope(x);
my=mdsmooth(y,100);
dmy=diff(my);
mdmy=mdsmooth(dmy,50);
end
```

distance.m

Purpose:

To compute distance measure between the signal and the four templates. Signals are 4096 points long.

Synopsis:

distance

Description:

Wavelet transform matrix of the signal and the templates are the inputs to the program. Program is written to work with the wavelet transform matrices produced by the subroutine "mapdn.m" The subroutine "mapdn.m" produces the magnitude square of the wavelet coefficients at each scale, "distance.m" needs the amplitudes of the wavelet coefficients. Thus, a modification to "mapdn.m" is essential prior to run "distance.m." mapdn.m and other functions called by mapdn.m can be found in (Newland, 1992) or in (Pitta, 1995). d1, d2, d3, d4 are the outputs of the program and they are distance measures between the signal and Template 1, Template 2, Template 3, Template 4, respectively at each scale.

Created by Yalcin Payal, November, 1995.

```
clear, clg
s1=input(' enter the WT of the signal to be identified = ');
kk1=input('enter the WT of the template 1= ');
mm1=input('enter the WT of the template 2= ');
tt1=input('enter the WT of the template 3= ');
vv1=input('enter the WT of the template 4= ');
for n=1:11
t=length(kk1(1,:))/(2^n);
col=n+2;
a=kk1(col,1:t:length(kk1(1,:)));
b=mm1(col,1:t:length(mm1(1,:)));
c=tt1(col,1:t:length(tt1(1,:)));
f=vv1(col,1:t:length(vv1(1,:)));
s=s1(col,1:t:length(s1(1,:)));
a=localext(a);
```

```

b=localext(b);
c=localext(c);
f=localext(f);
d=localext(s);
[temp1,i]=sort(a);
[temp2,j]=sort(b);
[temp3,k]=sort(c);
[temp4,k]=sort(f);
[x,m]=sort(d);
shift1=m(length(x))-i(length(a));
shift2=m(length(x))-j(length(b));
shift3=m(length(x))-k(length(c));
shift4=m(length(x))-q(length(f));
w1=[abs(m-i-shift1)];
w2=[abs(m-j-shift2)];
w3=[abs(m-k-shift3)];
w4=[abs(m-q-shift4)]; w1(find(w1==0))=ones(1,length(find(w1==0)));
w2(find(w2==0))=ones(1,length(find(w2==0)));
w3(find(w3==0))=ones(1,length(find(w3==0)));
w4(find(w4==0))=ones(1,length(find(w4==0)));
d1(n)=sum(sqrt(w1.*(temp1-x)^2));
d2(n)=sum(sqrt(w2.*(temp2-x)^2));
d3(n)=sum(sqrt(w3.*(temp3-x)^2));
d4(n)=sum(sqrt(w4.*(temp4-x)^2));
end

```

localext.m

Purpose:

To take the local extrema of the signal.

Synopsis:

localext.

Description:

LOCALEXT extracts the local extrema of a vector. LOCALEXT (X) returns a copy of vector X with all non extreme values set to 0.

[Y,K] = LOCALEXT (X) will also return the number of deleted samples of X.

```
function [ou,k]=localext(sg)
```

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Author: Sergio J. Garcia Galan, e-mail: Uvi_Wave@tsc.uvigo.es

```
l=length(sg);
sg=sg(:)';
ou=zeros(1,l);
ax=abs(sg);
MX = max(ax);
MX=MX+1;
ou(1)=sg(1);
ou(l)=sg(l);
k=0;
for i=2:l-1
    if (sg(i) > sg(i-1)) & (sg(i) > sg(i+1))
        ou(i)=sg(i);
        k=k+1;
    end
    if (sg(i) < sg(i-1)) & (sg(i) < sg(i+1))
        ou(i)=sg(i);
        k=k+1;
    end
end
end
k=l-k;
```

envelope.m

Purpose:

To take the envelope of the push-to-talk communication signals.

Synopsis:

envelope .

Description:

[Y,M]=ENVELOPE (x) returns the envelope , Y , and modulation index, M, of the AM signal in vector x.

Created by Dennis W. Brown, 1993.

Naval Postgraduate School, Monterey, CA.

May be freely distributed; not for use in commercial applications.

Part of the SPC Toolbox (Brown, 1995).

```
function [y,m] = envelope(x)
y = [];
if nargin == 1,
    error('envelope: One argument required...');
end
if min(size(x)) == 1,
    error('envelope: Input arg "x" must be a 1xN or Nx1 vector.');
```

```
end;
x = x(:);
y = abs(hilbert(x));
mmax = max(y);
mmin = min(y);
m = (mmax - mmin) / (mmax + mmin);
```

mdsmooth.m

Purpose:

To filter the envelope of the push-to-talk communication signals and the differential of the envelopes by median filtering.

Synopsis:

mdsmooth.

Description:

[y] = MDSMOOTH(X,L) smooths the input vector X using a median filter with a rectangular window of "L" samples.

MDSMOOTH is implemented as a mex function on some installations.

Created by LT Dennis W. Brown

Naval Postgraduate School, Monterey, CA

May be freely distributed; not for use in commercial applications.

Part of the SPC Toolbox (Brown, 1995).

```
function [y] = mdsmooth(x,L)
y = [];
if nargin == 2,
    error('mdsmooth: Invalid number of input arguments...');
end;
if min(size(x)) == 1,
    error('mdsmooth: Input arg "x" must be a 1xN or Nx1 vector.');
```

```
end;
x = x(:);
Ns = length(x);
y = zeros(Ns,1);
x = [zeros(L/2-1,1); x ; zeros ; zeros(L/2,1)];
for k=1:Ns
    y(k,1) = median(x(k:k+L-1,1));
end
```

LIST OF REFERENCES

- Akansu, A.N. and Haddad, R.A., *Multiresolution Signal Decomposition*, Academic Press, Inc., San Diego, CA, 1992.
- Aware, Inc., "Wavelet Signal Processing for Transient Feature Extraction," Technical Report, AD 920315, March, 1992.
- Brown, D.W., "SPC Toolbox: A Matlab-Based Software Package for Signal Analysis," Master's Thesis, Naval Postgraduate School, 1995.
- Burrus, C.S., and Gopinath, R.A., "Introduction to Wavelets and Wavelet Transforms," Tutorial #1, *IEEE ICASSP '93*, Minneapolis, MN, April, 1993.
- Carter, P.H. "Unknown Transient Detection using Wavelets," *SPIE Proceedings*, Vol. 2242, pp. 803-814, 1994.
- Ehara, N., Sasase, I., and Mori, S., "Weak Radar Signal Detection Based on Wavelet Transform," *IEEE Signal Processing*, Vol. 2, pp. 377-380, 1994.
- Gabor, D., "Theory of Communication," *Journal of IEEE*, Vol. 93, pp. 429-457, 1946.
- Hippenstiel, R., Wavelet Course Class Notes, Naval Postgraduate School, Monterey, CA, 1995.
- Mallat, S.G., and Zhoung, S., "Complete Signal Representation with Multiscale Edges," Technical Report 483, Courant Institute of Mathematical Sciences, New York University, December, 1989.
- Mathworks, MATLAB Reference Guide, August, 1992.
- Newland, D.E., "An Introduction to Random Vibrations," *Spectral and Wavelet Analysis*, Longman Scientific & Technical, John Wiley & Sons, Inc., New York, 1992.
- Pitas, I., and Venetsanopoulos, A.N., *Nonlinear Digital Filters*, Kluwer Academic Publishers, Boston, 1990.

Pitta, J.R., "Transient Detection Using Wavelets," Master's Thesis, Naval Postgraduate School, Monterey, CA, 1995.

Shensa, M.J., "Discrete Wavelet Transform: Wedding A Trous and Mallat Algorithms," *IEEE Transactions on Signal Processing*, Vol 40, p 2464- 2482, 1992.

Strang, G., *Linear Algebra and Its Applications*, Academic Press, New York, 1976.

Vetterli, M., and Kovacevic, J., *Wavelets and Subband Coding*, Prentice Hall, Inc., Englewood Cliffs, NJ, 1995.

Weiss, L.G., "Wavelets and Wideband Correlation Process," *IEEE Signal Processing*, pp.13-32, January, 1994.

Young, R.K., *Wavelet Theory and its Applications*, Kluwer Academic Publishers, Boston, 1993.

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